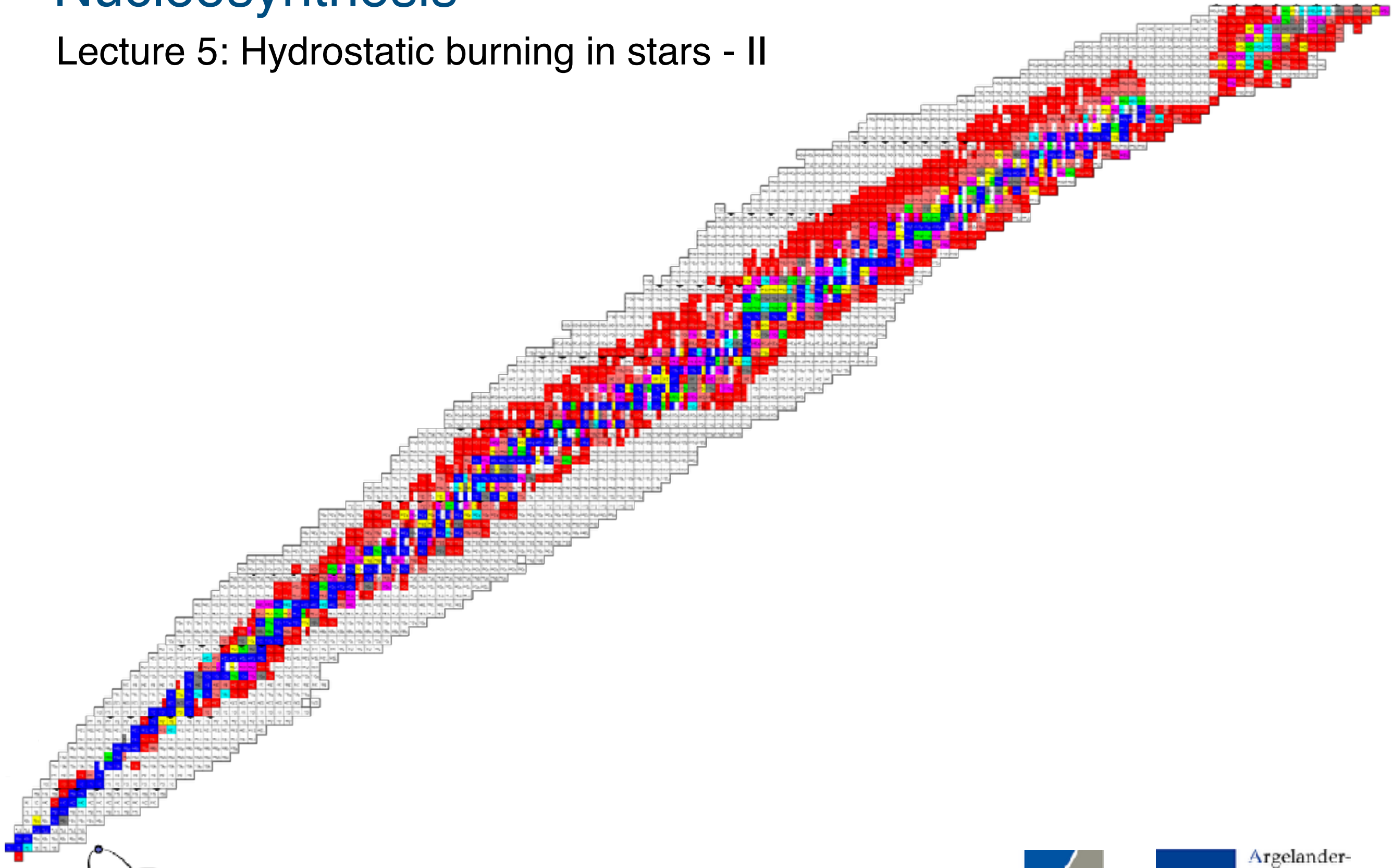


Nucleosynthesis

Lecture 5: Hydrostatic burning in stars - II



Overview

- **Lecture 1:** Introduction & overview
- **Lecture 2:** Thermonuclear reactions
- **Lecture 3:** Big-bang nucleosynthesis
- **Lecture 4:** Thermonuclear reactions inside stars — I (H-burning)
- **Lecture 5:** Thermonuclear reactions inside stars — II (advanced burning)
- **Lecture 6:** Neutron-capture and supernovae — I
- **Lecture 7:** Neutron-capture and supernovae — II
- **Lecture 8:** Thermonuclear supernovae
- **Lecture 9:** Li, Be and B
- **Lecture 10:** Galactic chemical evolution and relation to astrobiology

Paper presentations I

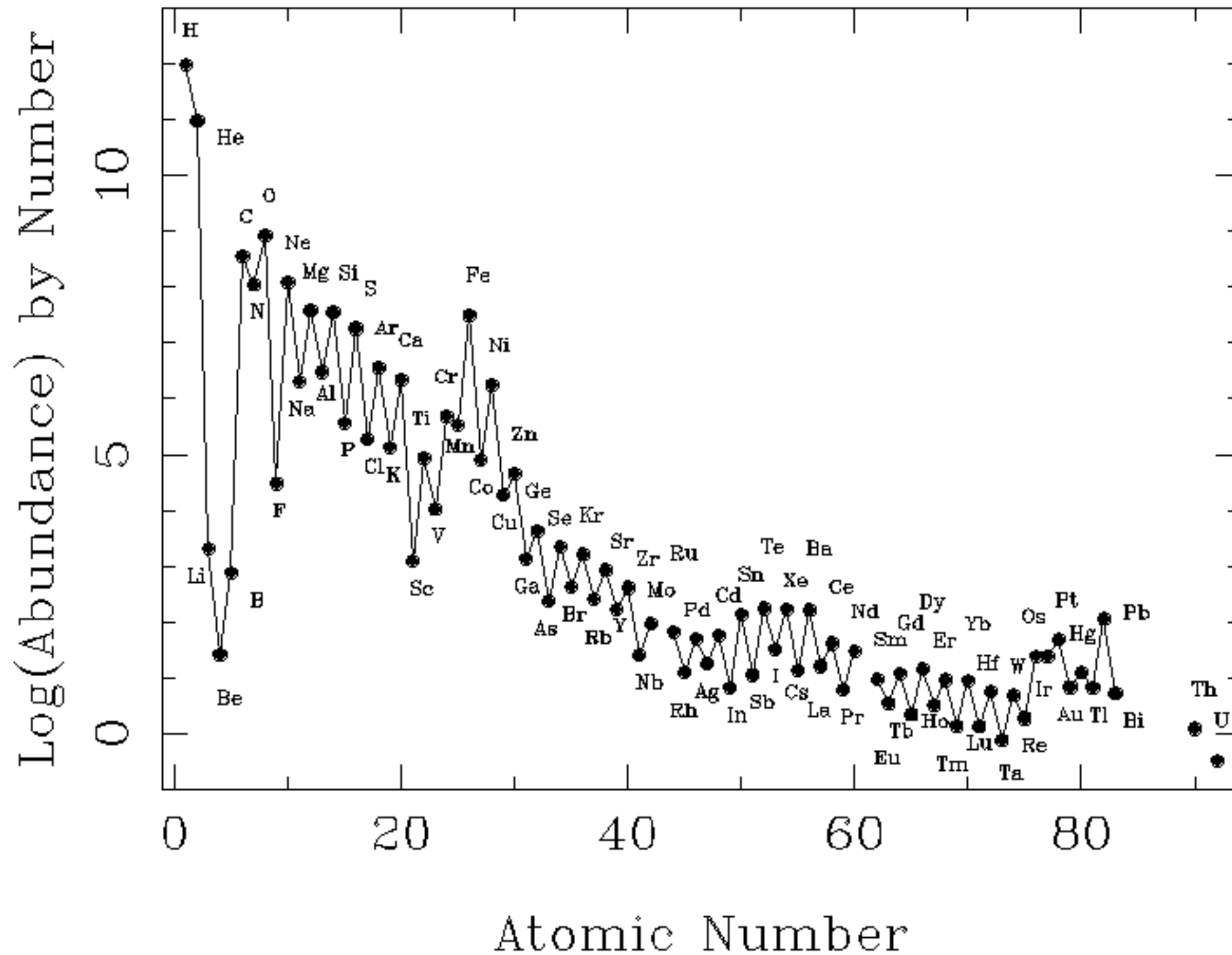
June 21

Paper presentations II

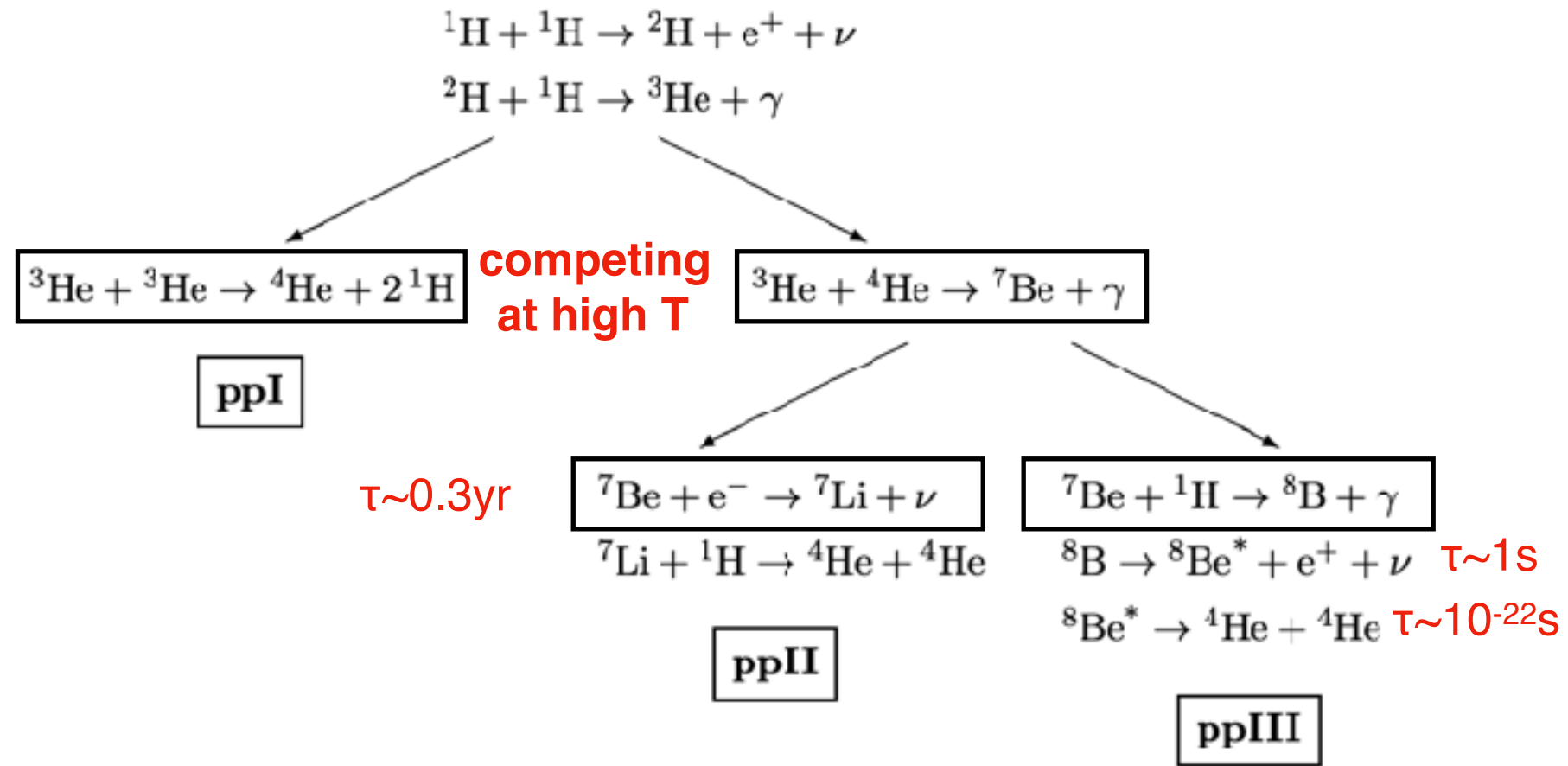
June 28

Overview of previous lectures

Logarithmic SAD Abundances: $\text{Log}(H) = 12.0$

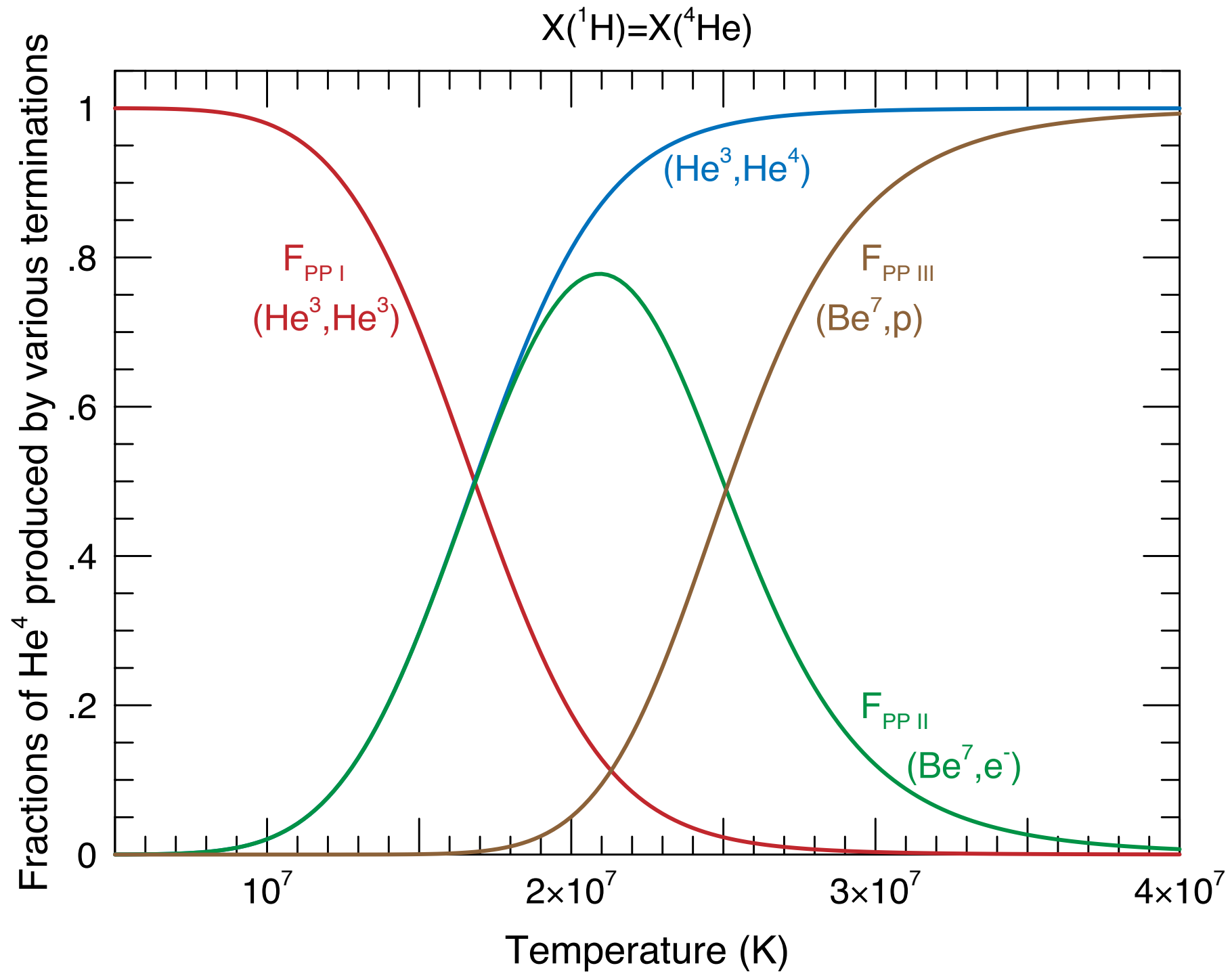


pp chains



reaction	Q (MeV)	$\langle E_\nu \rangle$ (MeV)	$S(0)$ (keV barn)	dS/dE (barn)	τ (yr)
${}^1\text{H}(p, e^+ \nu){}^2\text{H}$	1.442	0.265	3.94×10^{-22}	4.61×10^{-24}	10^{10}
${}^2\text{H}(p, \gamma){}^3\text{He}$	5.493		2.5×10^{-4}	7.9×10^{-6}	10^{-8}
${}^3\text{He}({}^3\text{He}, 2p){}^4\text{He}$	12.860		5.18×10^3	-1.1×10^1	10^5
${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$	1.587		5.4×10^{-1}	-3.1×10^{-4}	10^6
${}^7\text{Be}(e^-, \nu){}^7\text{Li}$	0.862	0.814			10^{-1}
${}^7\text{Li}(p, \alpha){}^4\text{He}$	17.347		5.2×10^1	0	10^{-5}
${}^7\text{Be}(p, \gamma){}^8\text{B}$	0.137		2.4×10^{-2}	-3×10^{-5}	10^2
${}^8\text{B}(e^+ \nu){}^8\text{Be}^*(\alpha){}^4\text{He}$	18.071	6.710			10^{-8}

pp chains



The CNO cycles

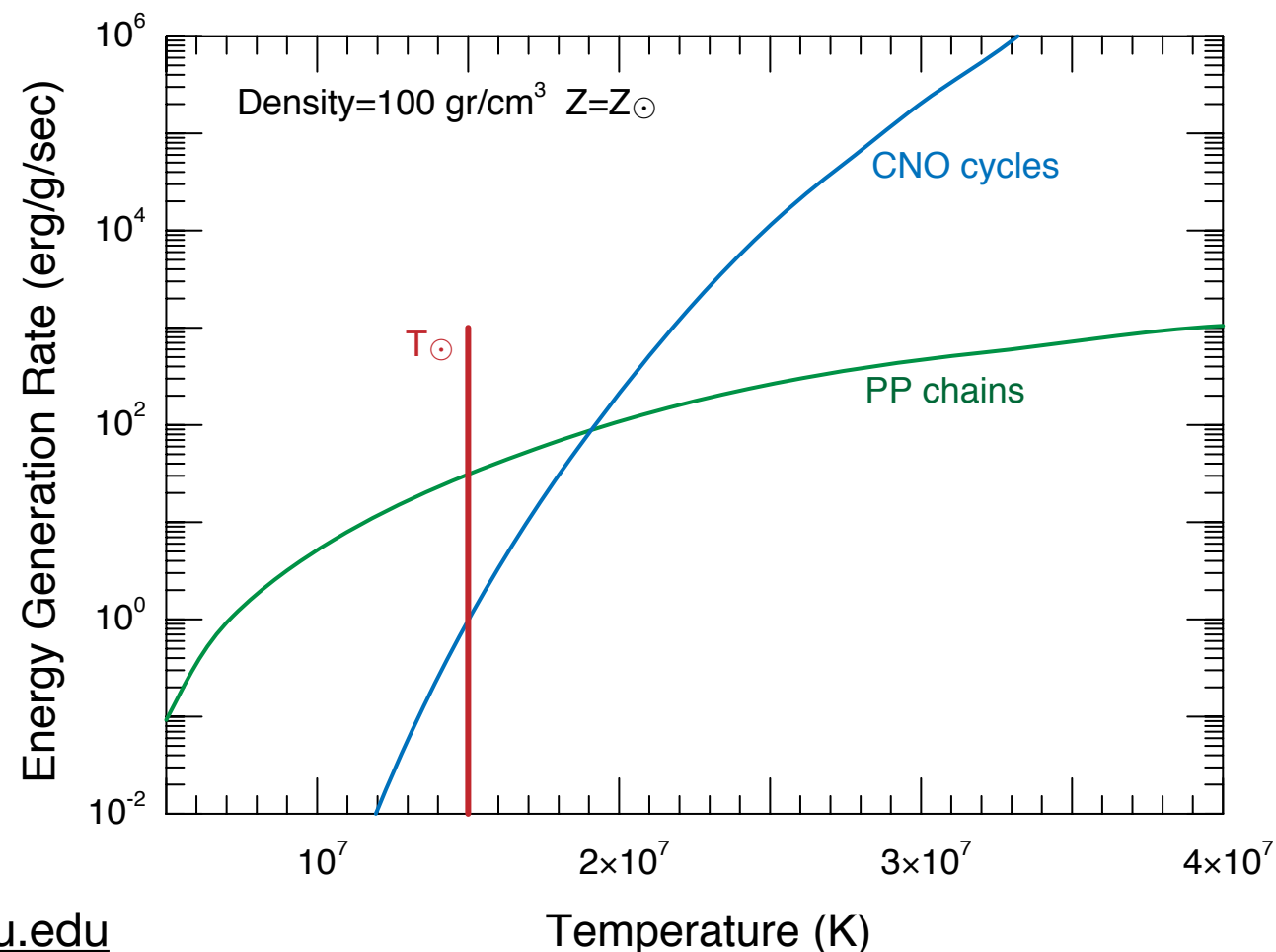
The pp chains can operate in pure H/He gasses to synthesise ^4He from H. However, stars like our Sun also have a healthy admixture of heavier elements. Therefore, it becomes necessary to consider other reactions as possible sources of energy, even at “low” central temperatures typical on the main sequence.

Since lifetimes rise rapidly with increasing coulomb barrier, the best candidates must have charges such that the product Z_1Z_2 is as small as possible

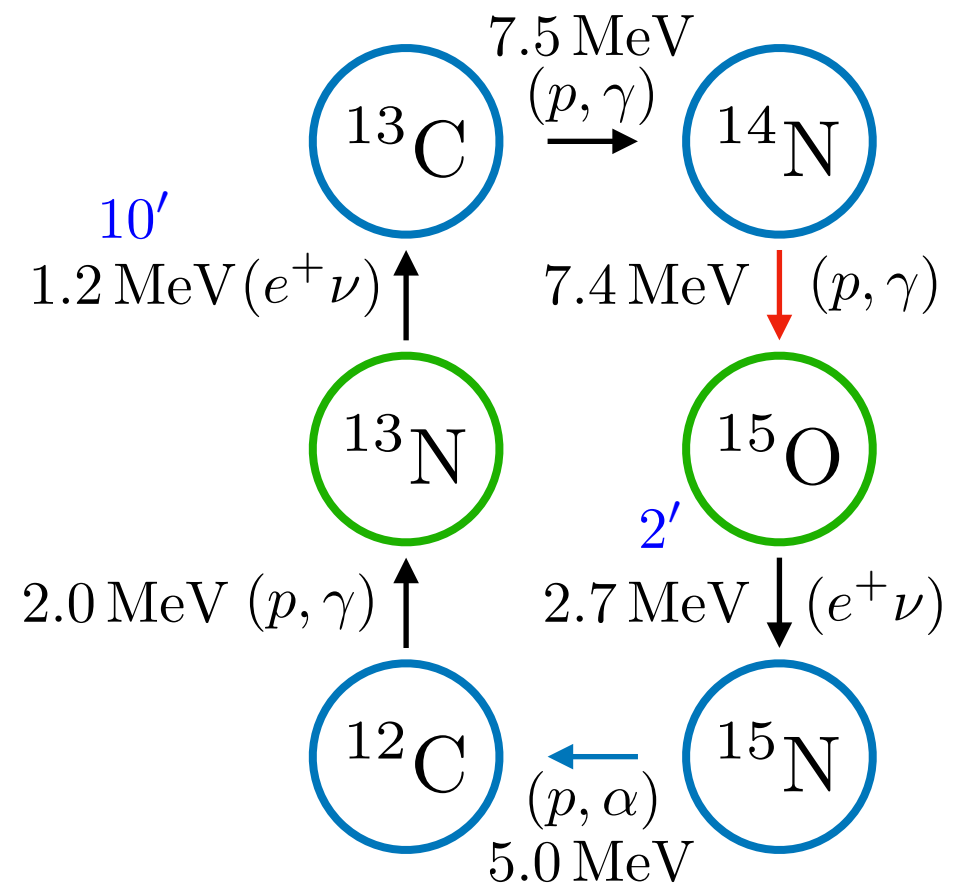
A series of such reactions, involving C and N nuclei was discovered by **Bethe & von Weizsäcker (1938)**

This series has the property that the CN nuclei serve only as catalysts for the conversion of H to He

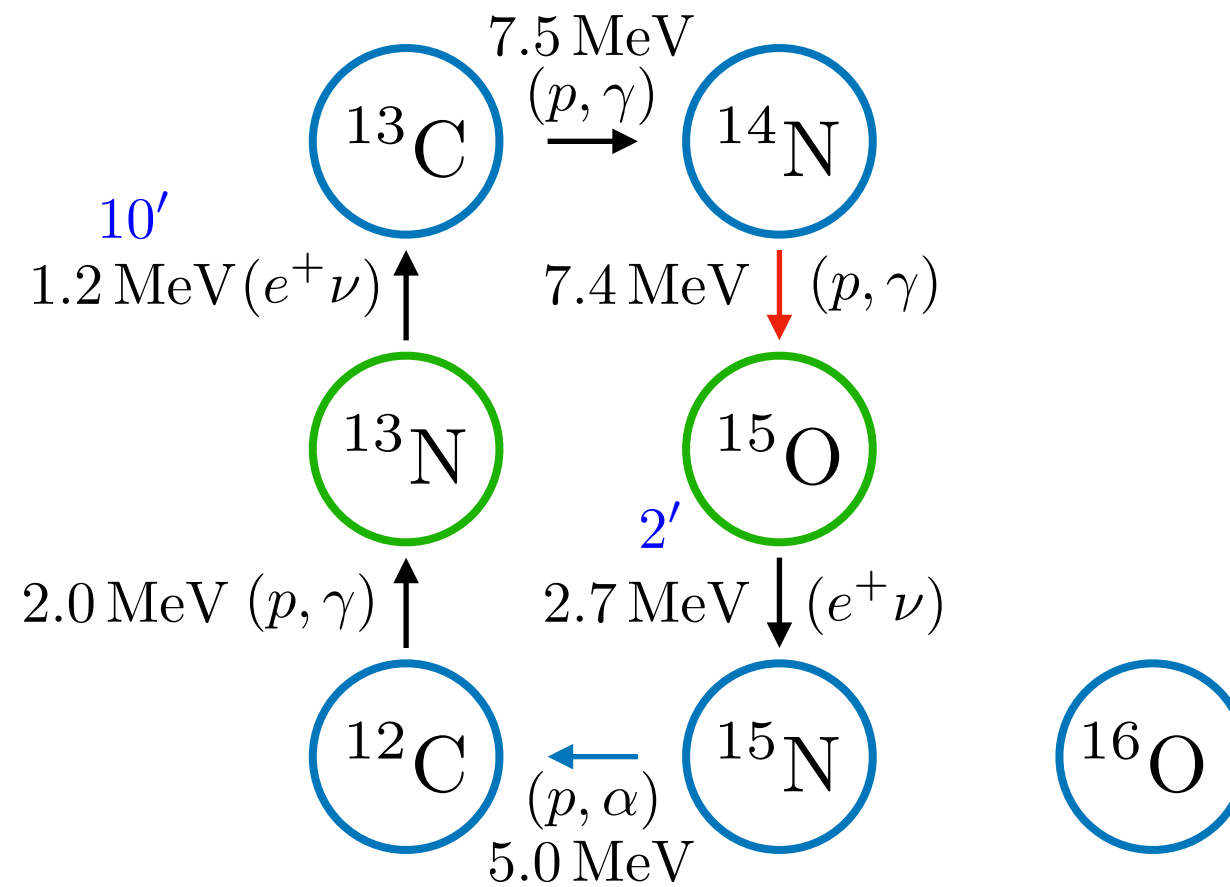
For solar composition, the CN cycle produces more energy than the pp-chains for $T_6 > \sim 20$,



The CNO reaction flow and branching



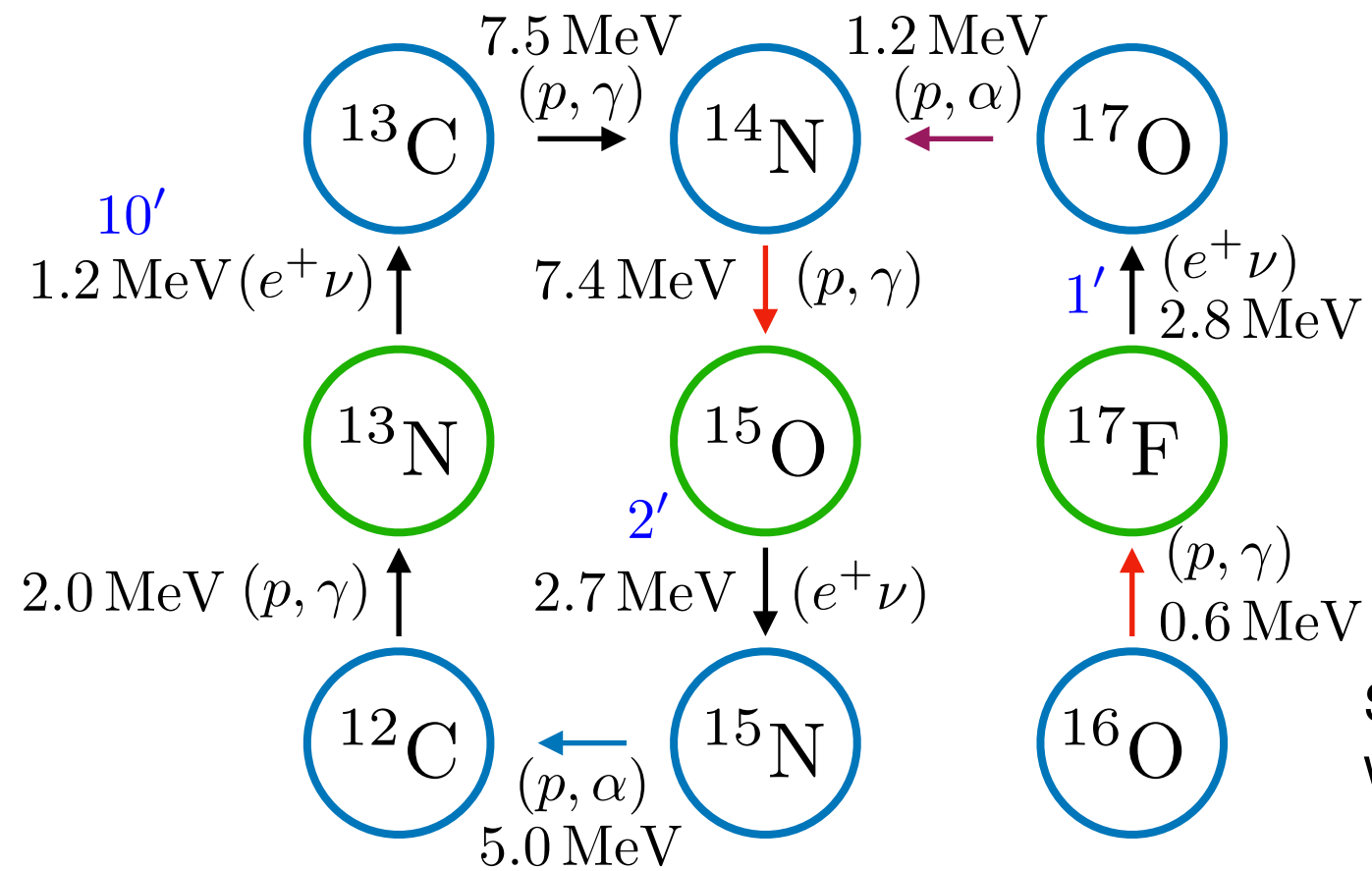
The CNO reaction flow and branching



Second cycle initiated
when oxygen is present



The CNO reaction flow and branching

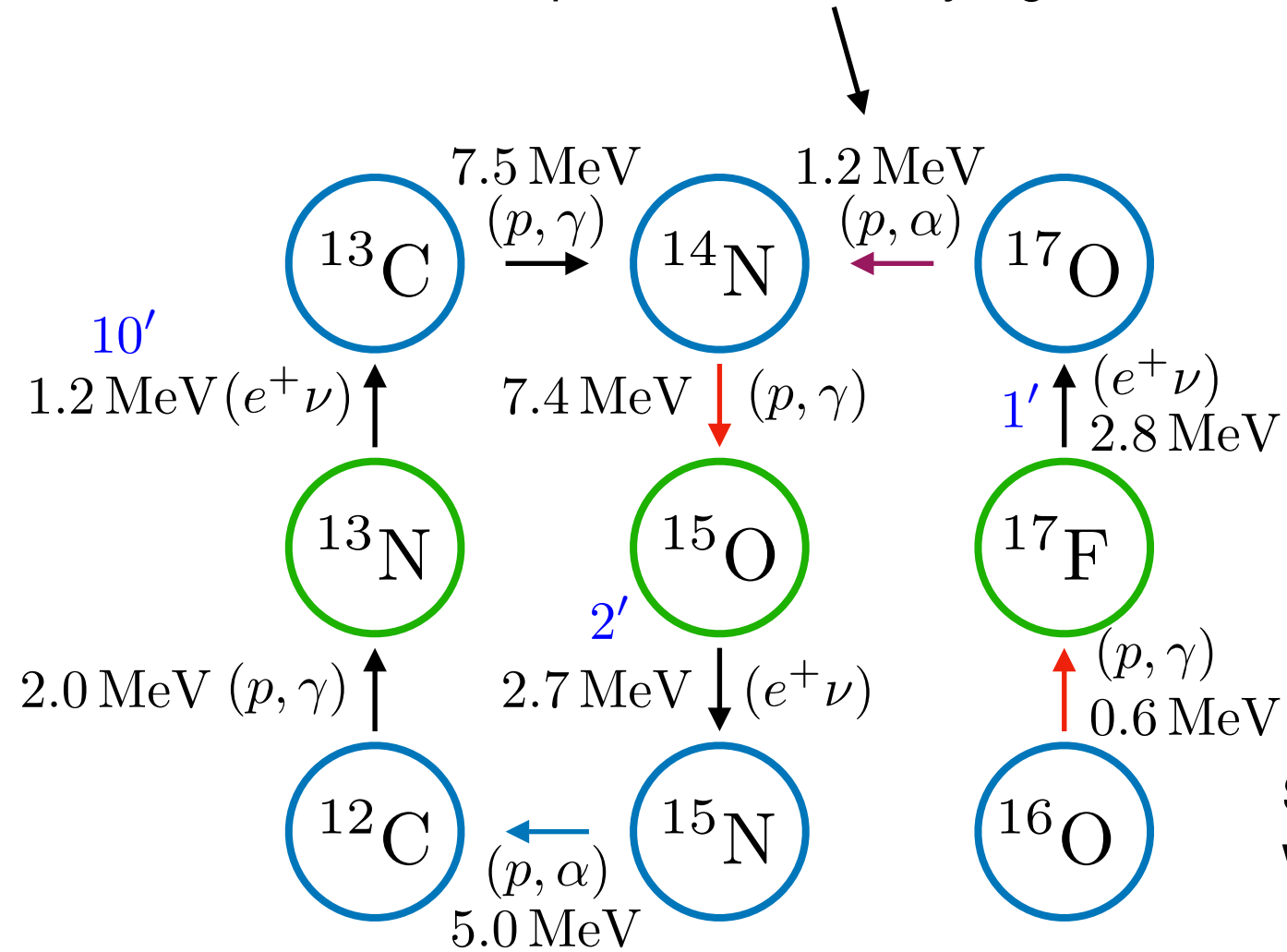


Second cycle initiated when oxygen is present



The CNO reaction flow and branching

This is the first resonant reaction that we encounter! The resonance becomes important at relatively high T

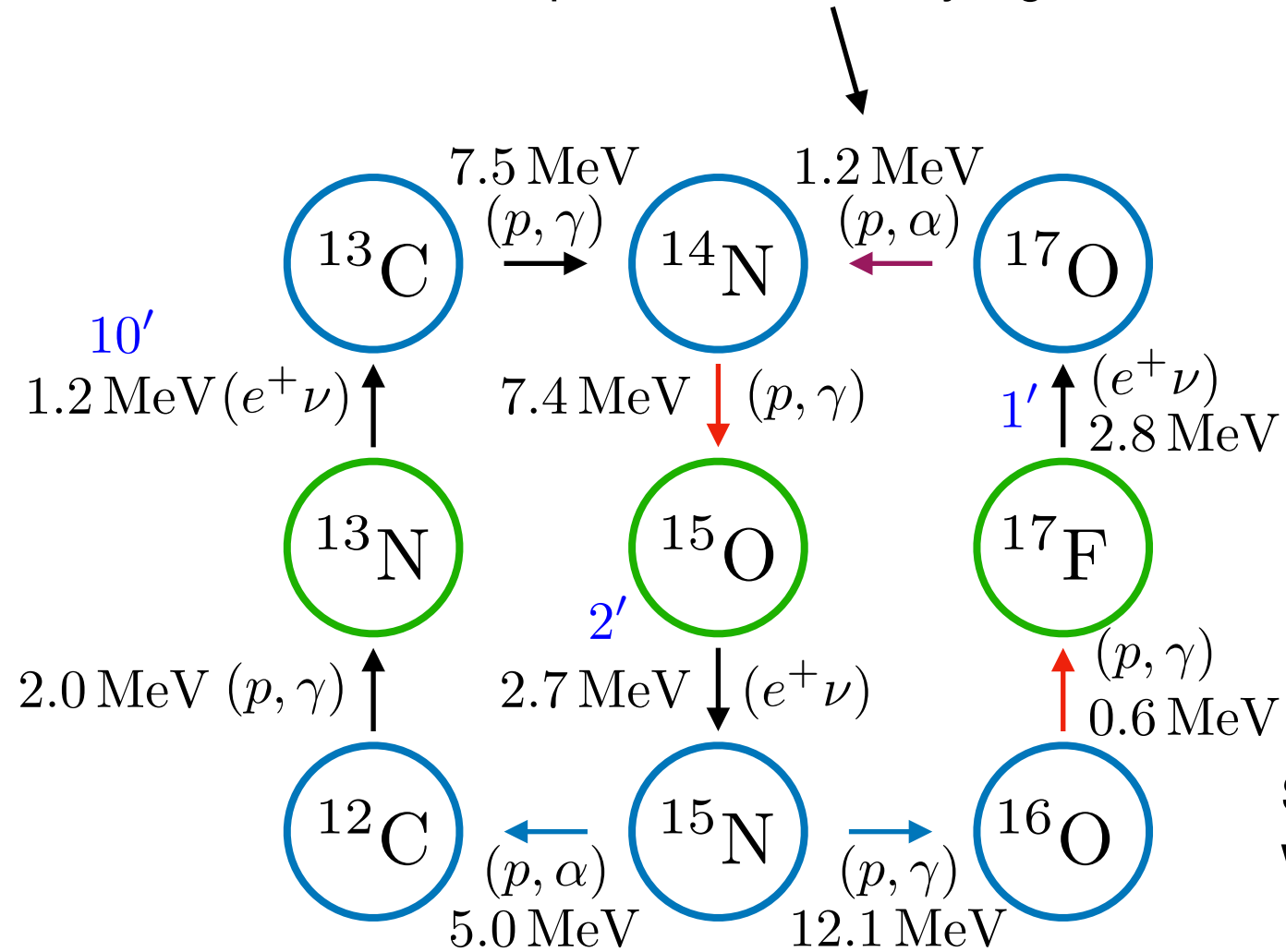


Second cycle initiated when oxygen is present



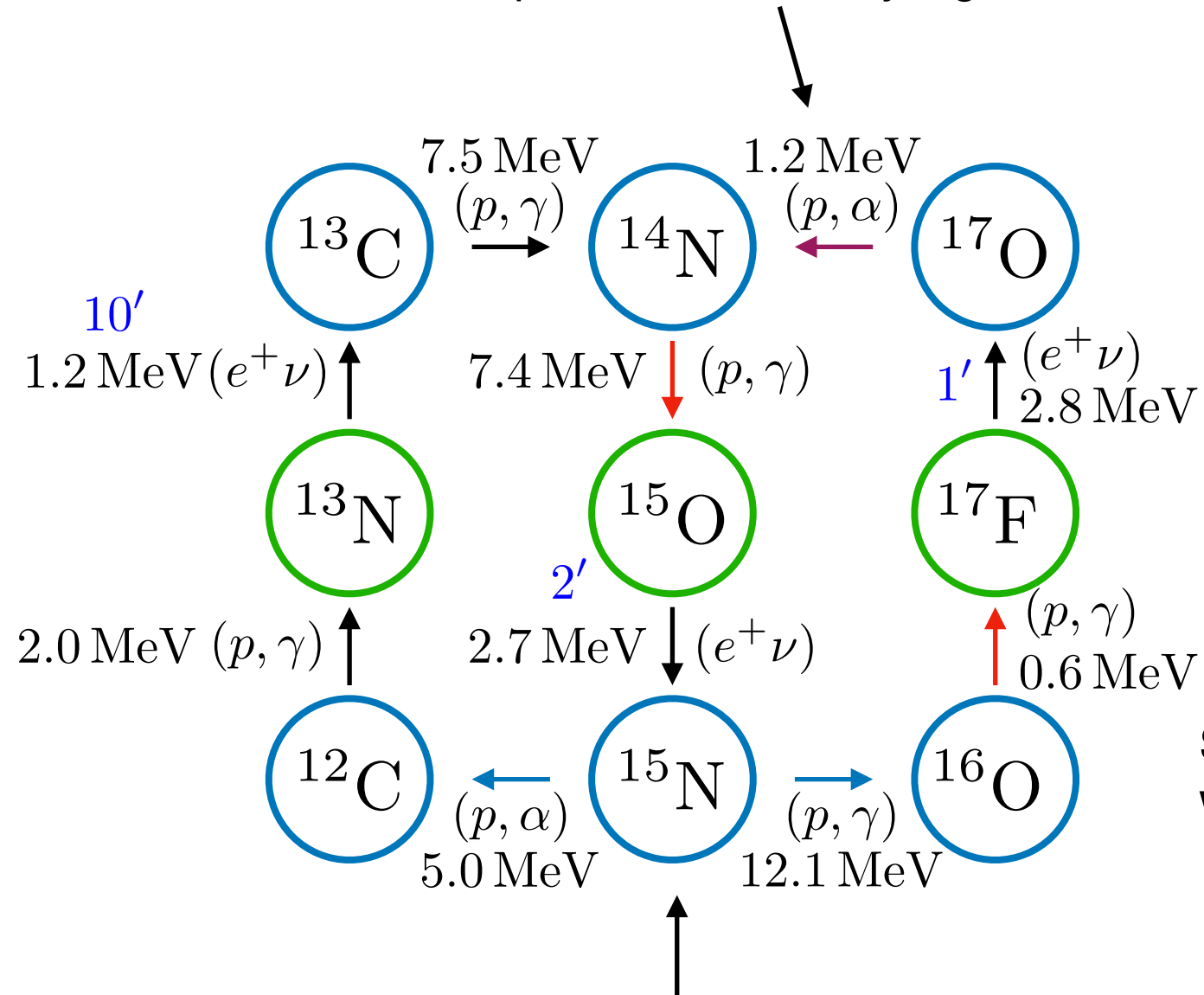
The CNO reaction flow and branching

This is the first resonant reaction that we encounter! The resonance becomes important at relatively high T



The CNO reaction flow and branching

This is the first resonant reaction that we encounter! The resonance becomes important at relatively high T



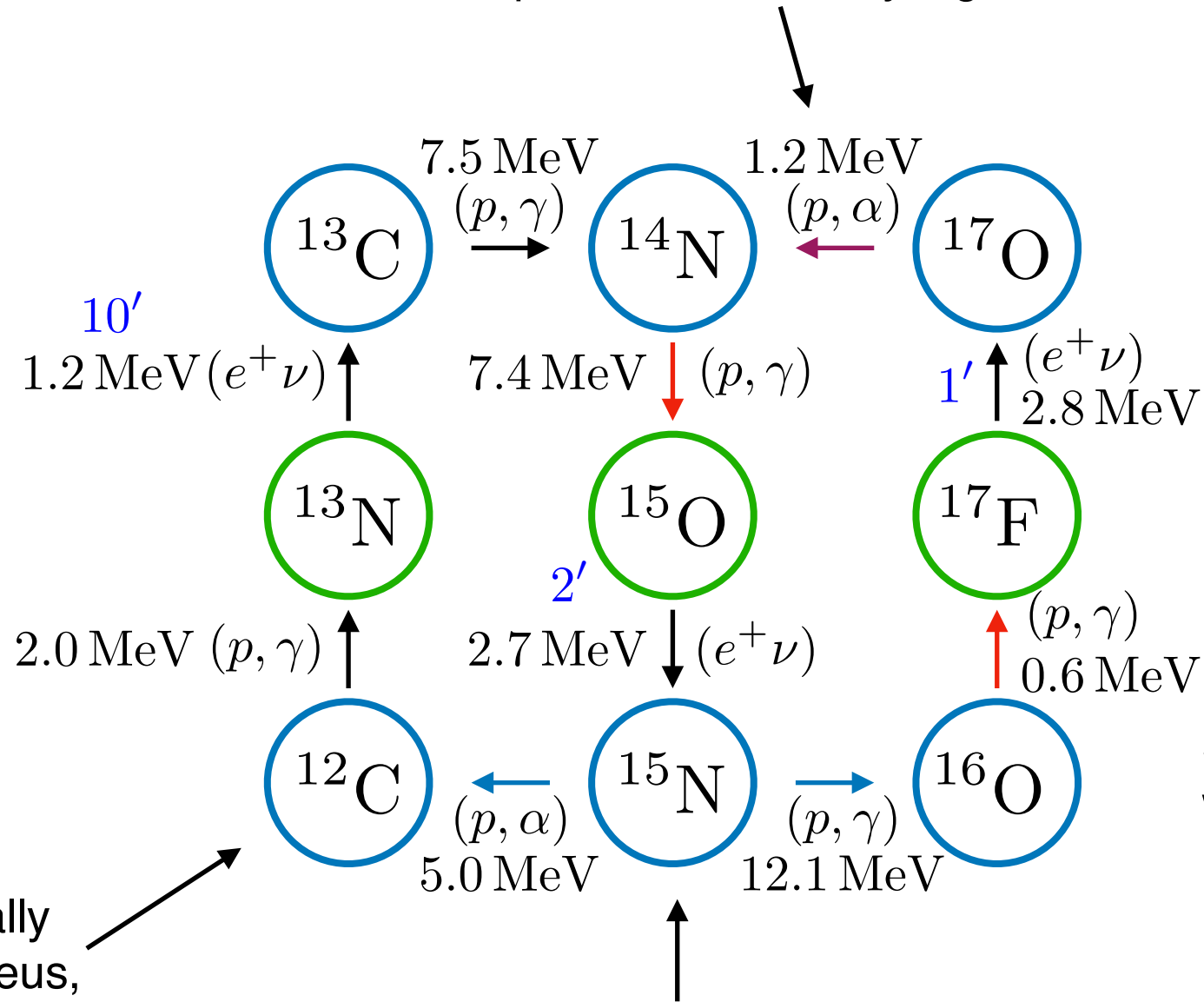
Second cycle initiated when oxygen is present



~4 gamma decays for every ~10000 alpha decays

The CNO reaction flow and branching

This is the first resonant reaction that we encounter! The resonance becomes important at relatively high T



Likely the most initially abundant seed nucleus, followed by ^{16}O

~4 gamma decays for every ~10000 alpha decays

Second cycle initiated when oxygen is present



The CNO bi-cycles

Just like in pp reactions, the key to understanding the CNO reactions lies in appreciating the lifetimes of nuclei against protons. Recall that for non-resonant reactions, the lifetime of a nucleus against proton capture is:

$$\frac{1}{\tau_p(X)} = 2.4 \times 10^{16} \rho X_H f S_0 \left[\frac{(A_X + 1) Z_X}{A_X} \right]^{1/3} T_6^{-2/3} \left(1 + \frac{5}{12 B T_6^{1/3}} \right) \exp(-B T_6^{-1/3}) \text{ yr}^{-1}$$

$$\text{where } S_0 = S(0) + \frac{dS}{dE} \left[1.22 \left(\frac{Z_X A_X T_6}{A_X + 1} \right)^{1/3} + 0.072 T_6 \right]; \quad B = 42.48 (Z_X^2 A_X)^{1/3}$$

reaction	Q (MeV)	$\langle E_\nu \rangle$ (MeV)	$S(0)$ (MeV barn)	dS/dE (barn)	τ (yr)
$^{12}\text{C}(p, \gamma)^{13}\text{N}$	1.944		1.45×10^{-3}	2.45×10^{-3}	6.6×10^3
$^{13}\text{N}(e^+ \nu)^{13}\text{C}$	2.220	0.707			863 s
$^{13}\text{C}(p, \gamma)^{14}\text{N}$	7.551		5.50×10^{-3}	1.34×10^{-2}	1.6×10^3
$^{14}\text{N}(p, \gamma)^{15}\text{O}$	7.297		3.32×10^{-3}	-5.91×10^{-3}	9.3×10^5
$^{15}\text{O}(e^+ \nu)^{15}\text{N}$	2.754	0.997			176 s
$^{15}\text{N}(p, \alpha)^{12}\text{C}$	4.965		7.80×10^1	3.51×10^2	3.5×10^1
$^{15}\text{N}(p, \gamma)^{16}\text{O}$	12.127		6.4×10^{-2}	3×10^{-2}	3.9×10^4
$^{16}\text{O}(p, \gamma)^{17}\text{F}$	0.600		9.4×10^{-3}	-2.3×10^{-2}	7.1×10^7
$^{17}\text{F}(e^+ \nu)^{17}\text{O}$	2.761	0.999			93 s
$^{17}\text{O}(p, \alpha)^{14}\text{N}$	1.192		resonant reaction		1.9×10^7

WARNING: Most of these are outdated. For more recent values see Adelberger et al. 2011

CNO cycles approaching equilibrium

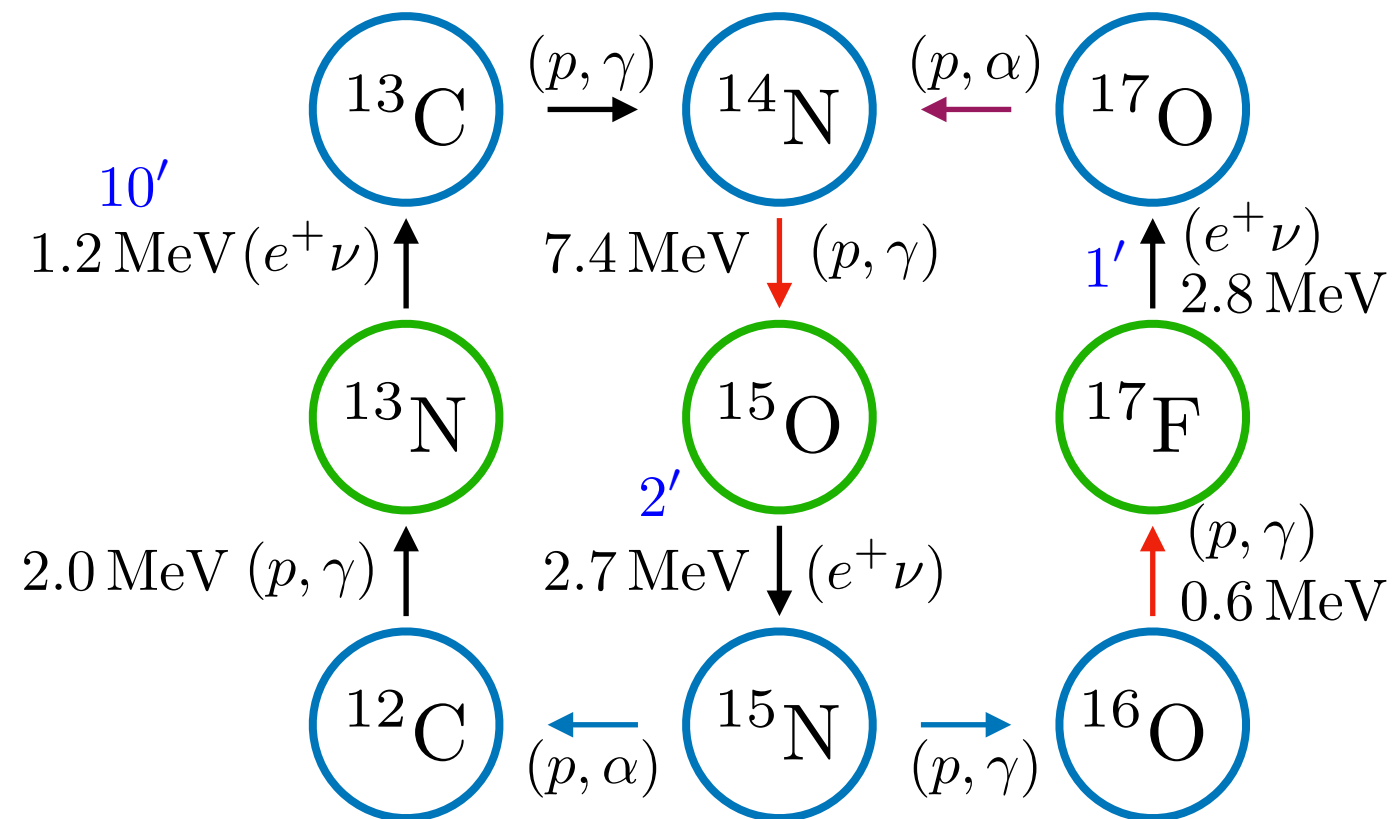
$$\tau_p(15) \ll \tau_p(13) \ll \tau_p(12) \ll \tau_p(14) : 35 \text{ yr} : 1600 \text{ yr} : 6600 \text{ yr} : 9 \times 10^5 \text{ yr}$$

To follow the abundance evolution, one needs to consider the differential equations for all reactants. Fortunately, some simplifications can be made due to the vastly different lifetimes.

It would help if the two cycles were independent. This would be the case if $X(^{16}\text{O})=0$ and $^{15}\text{N}(p,\alpha)^{12}\text{C}$ only. Turns out we can treat them independently anyways!

Why?

In cycle I, the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction determines the cycle speed. A ^{15}N nucleus is produced every τ_{14} years. One in every ~ 2500 of these nuclei go to the second cycle. Similarly a ^{14}N is produced by cycle II every τ_{16} yr. Both these times are much longer than the τ_{14}



CNO cycles approaching equilibrium

Consequences

Cycle I reaches equilibrium much faster than Cycle II, and thus it can be treated independently.

When analysing Cycle II we can assume that ^{14}N and ^{15}N have reached their equilibrium abundances.

The ODEs for Cycle I are as follows. The β^+ decays are taken to be instantaneous since the products reach their equilibrium abundances very fast, in times of order τ_β (sec).

$$\frac{d^{15}\text{N}}{dt} = H \left(^{14}\text{N} \langle \sigma v \rangle_{14} - ^{15}\text{N} \langle \sigma v \rangle_{15} \right)$$

$$\frac{d^{12}\text{C}}{dt} = H \left(^{15}\text{N} \langle \sigma v \rangle_{15} - ^{12}\text{C} \langle \sigma v \rangle_{12} \right)$$

$$\frac{d^{13}\text{C}}{dt} = H \left(^{12}\text{C} \langle \sigma v \rangle_{12} - ^{13}\text{C} \langle \sigma v \rangle_{13} \right)$$

$$\frac{d^{14}\text{N}}{dt} = H \left(^{13}\text{C} \langle \sigma v \rangle_{13} - ^{14}\text{N} \langle \sigma v \rangle_{14} \right)$$

CNO cycles approaching equilibrium

Consequences

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$$\begin{aligned} \frac{d^{15}\text{N}}{dt} &= H \left(^{14}\text{N} \langle \sigma v \rangle_{14} - ^{15}\text{N} \langle \sigma v \rangle_{15} \right) \longrightarrow \left(\frac{^{15}\text{N}}{^{14}\text{N}} \right)_e = \frac{\tau_{15}}{\tau_{14}} = 4 \times 10^{-5} \\ \frac{d^{12}\text{C}}{dt} &= H \left(^{15}\text{N} \langle \sigma v \rangle_{15} - ^{12}\text{C} \langle \sigma v \rangle_{12} \right) \\ \frac{d^{13}\text{C}}{dt} &= H \left(^{12}\text{C} \langle \sigma v \rangle_{12} - ^{13}\text{C} \langle \sigma v \rangle_{13} \right) \\ \frac{d^{14}\text{N}}{dt} &= H \left(^{13}\text{C} \langle \sigma v \rangle_{13} - ^{14}\text{N} \langle \sigma v \rangle_{14} \right) \end{aligned}$$

In times of order τ_{15} (years)

CNO cycles approaching equilibrium

Consequences

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On earth: $\left(\frac{^{15}\text{N}}{^{14}\text{N}} \right)_{\text{earth}} \simeq 4 \times 10^{-3}$ What could be the reason for the discrepancy?

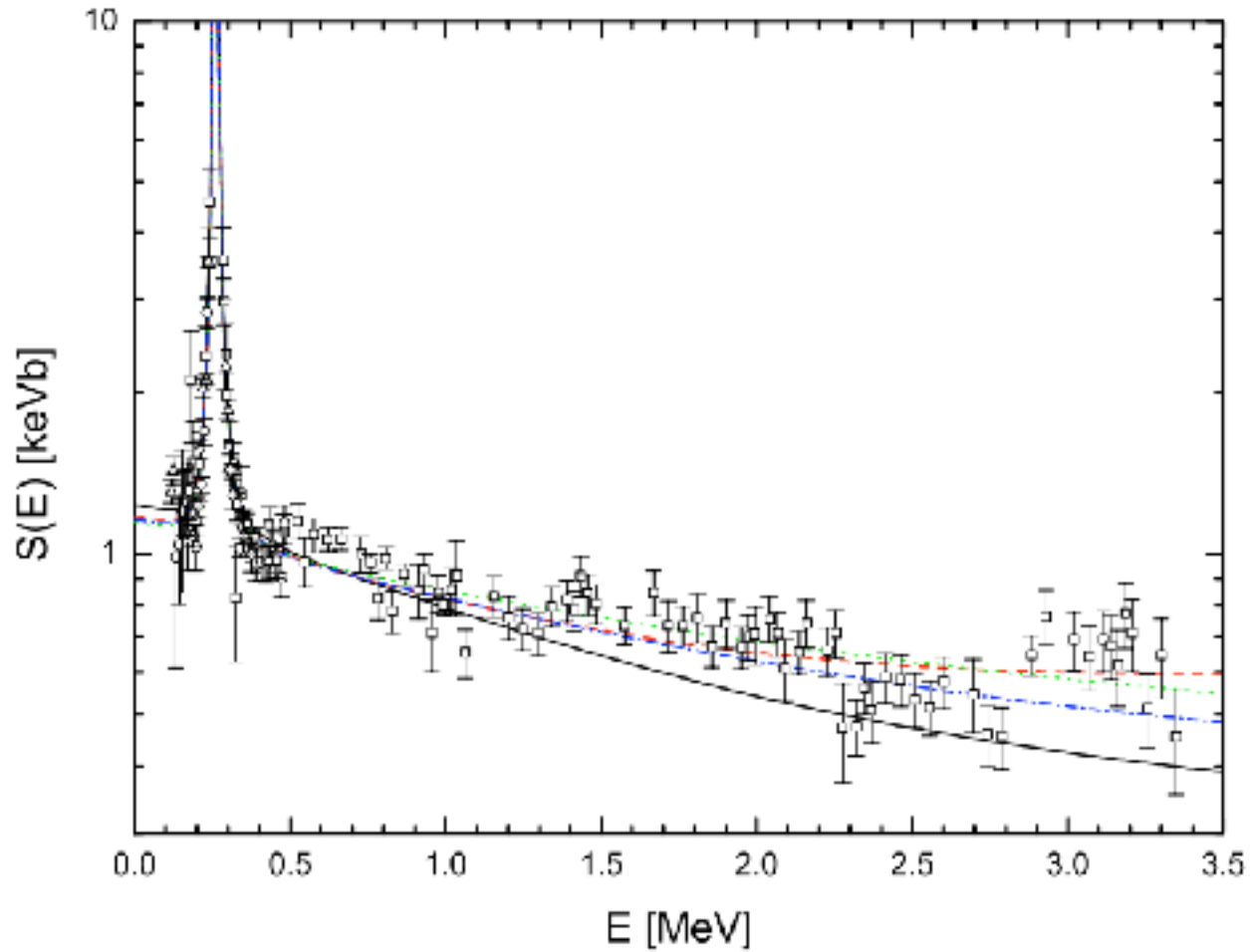
CNO cycles approaching equilibrium

One possible reason could be a hidden resonance in the bottleneck reaction $^{14}\text{N}(p,\gamma)^{15}\text{O}(e^+\nu)^{15}\text{N}$, producing more ^{15}N

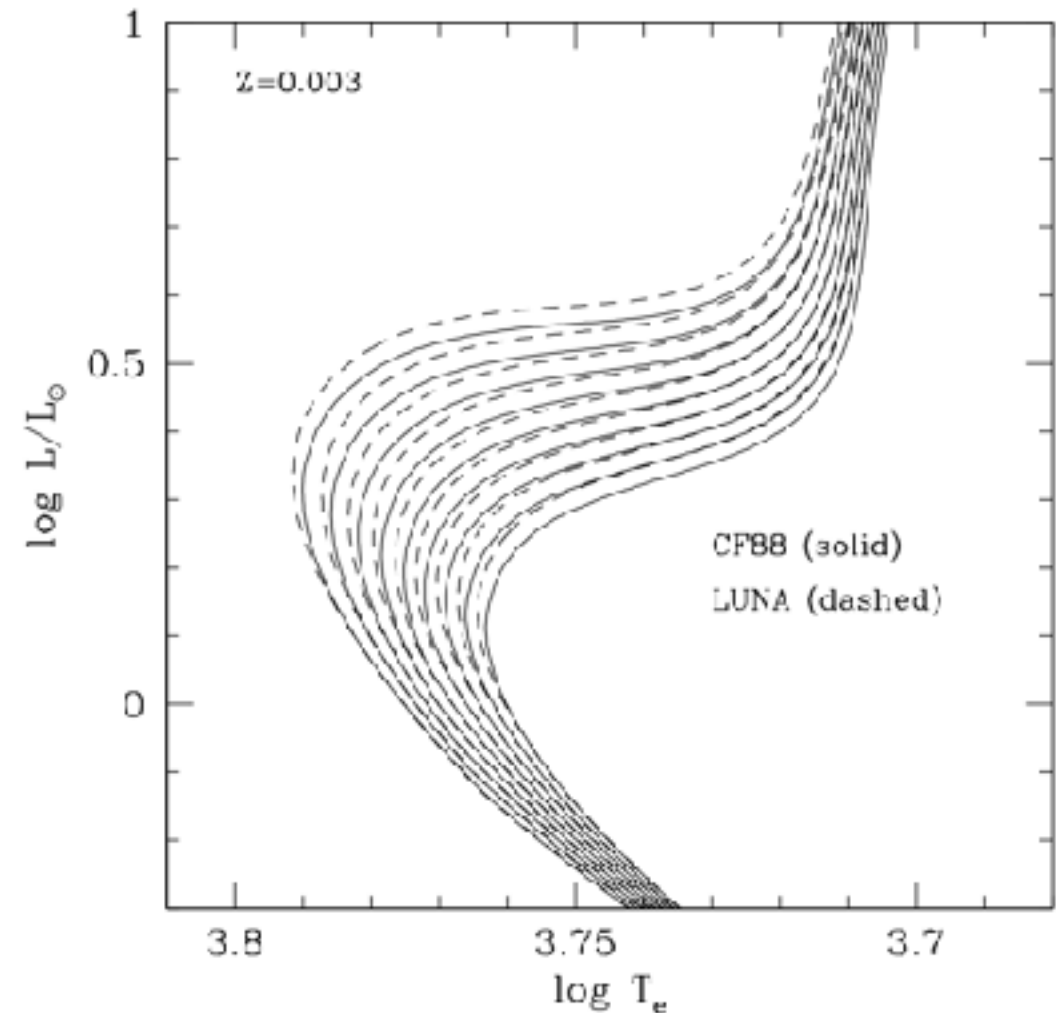
This would also affect the age determination of GCs via the MS turn-off

E_{CM} [keV]	E_x [keV]	J^π
2187	9484	$3/2^+$
987	8284	$3/2^+$
7297	7556	$1/2^+$
259	7276	$7/2^-$
$^{14}\text{N} + \text{p}$	6859	$5/2^+$
-506	6791	$3/2^+$
-1121	6172	$3/2^-$
-2117	5241	$5/2^-$
	5181	$1/2^-$
	0	$1/2^-$

^{15}O

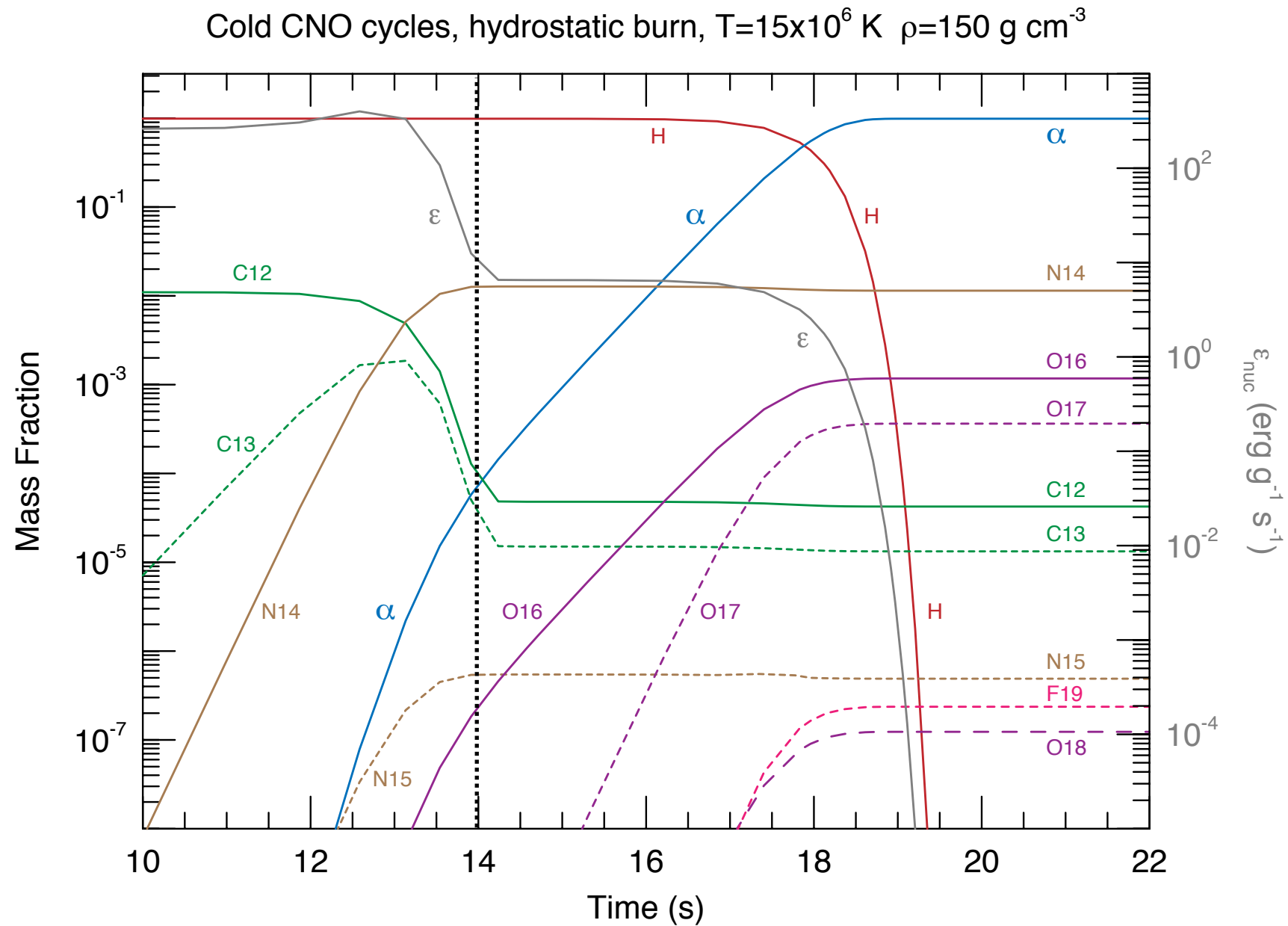


No evidence for resonances at low energies.
Some at high energy could influence explosive burning



CNO cycles approaching equilibrium

Similar analysis can be performed for the remaining three ODEs (see CLAYTON for a detailed discussion). Once CNO-I reaches equilibrium, the same analysis can follow for CNO-II, adopting the CNO-I equilibrium abundances



CNO-I: C12 is the last to reach equilibrium. N14 is strongly produced

CNO-II: O16 is the last to reach equilibrium. N14 and O16 produced

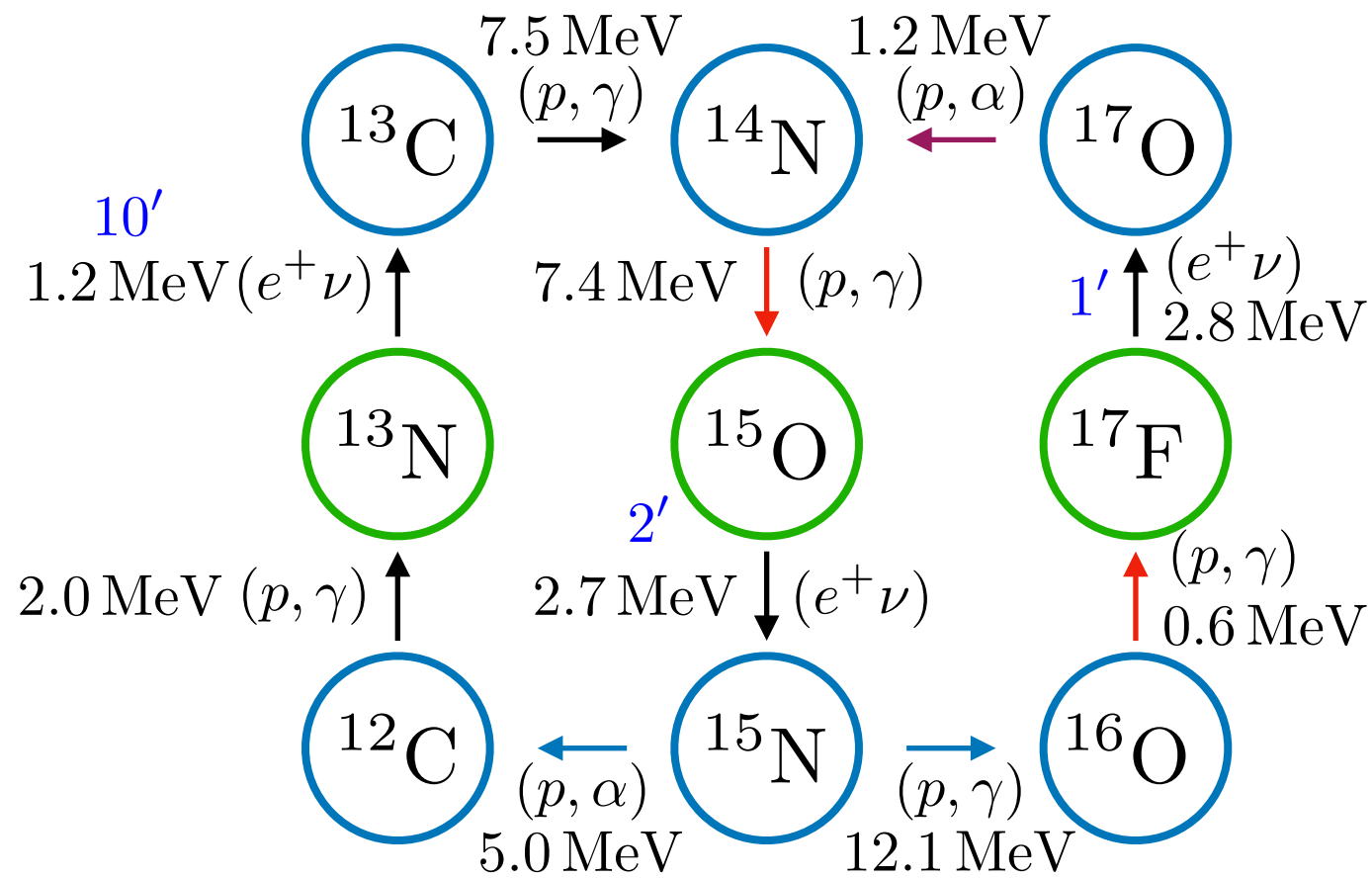
CNO cycles approaching equilibrium

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	^{12}C	^{13}C	^{14}N	^{15}N	^{16}O	^{17}O	^{18}O	^{19}F
equil. mass fraction ($Z = 2\%$, $T = 3 \cdot 10^7 \text{ K}$)	10^{-4}	$6 \cdot 10^{-5}$	10^{-2}	$3 \cdot 10^{-7}$	$3 \cdot 10^{-4}$	$4 \cdot 10^{-6}$	10^{-9}	10^{-9}
solar mass fraction	$3.5 \cdot 10^{-3}$	$4 \cdot 10^{-5}$	10^{-3}	$4 \cdot 10^{-6}$	10^{-2}	$4 \cdot 10^{-6}$	$2 \cdot 10^{-5}$	$4 \cdot 10^{-7}$
ratio	$\frac{1}{30}$	1.5	10	$\frac{1}{10}$	$\frac{1}{30}$	1	$\frac{1}{20\,000}$	$\frac{1}{400}$

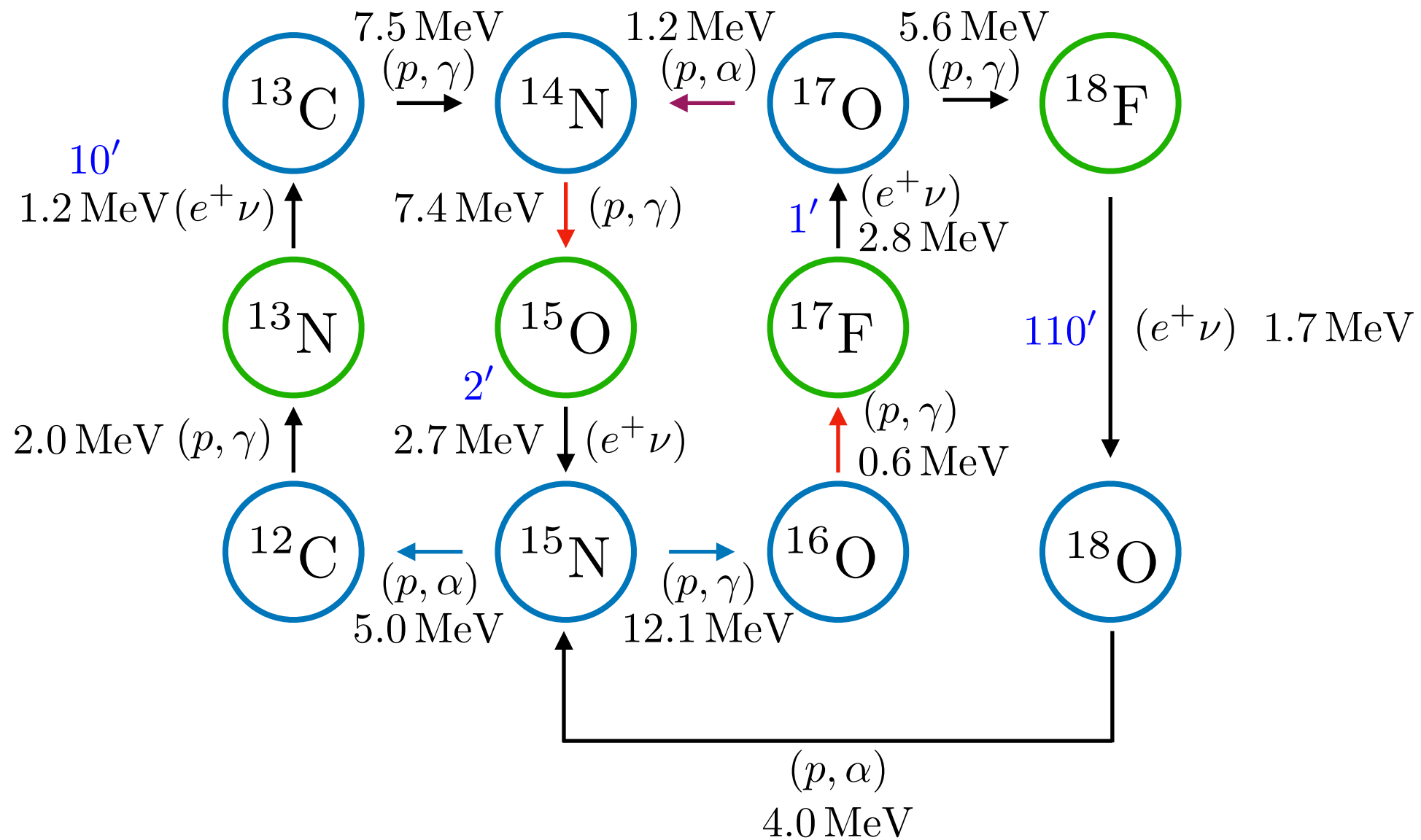
More CNO branches. HCNO and breakout

Is there a limit to how fast the CNO cycles can run? At high temperatures, some reactions become faster than beta decays



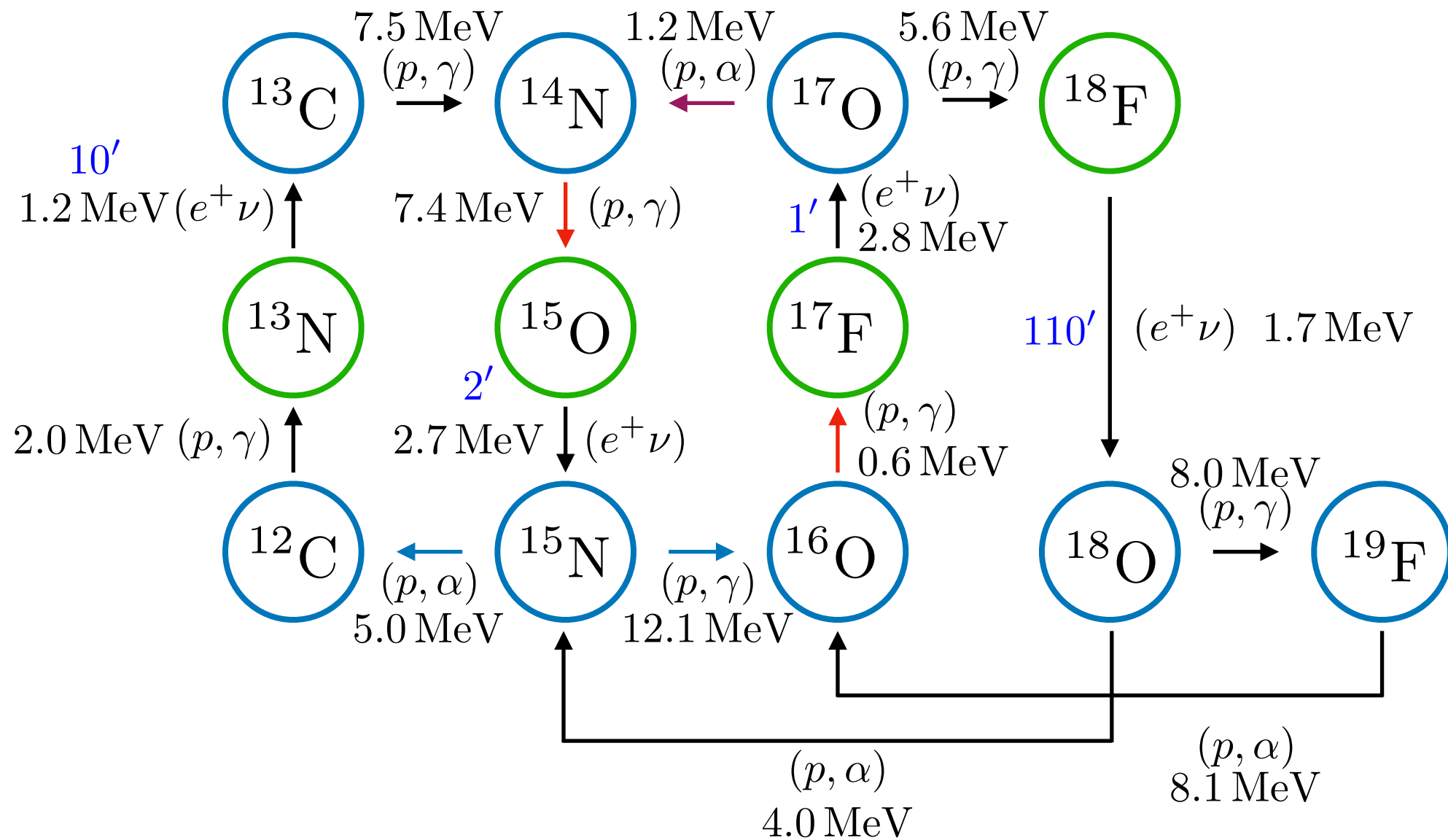
More CNO branches. HCNO and breakout

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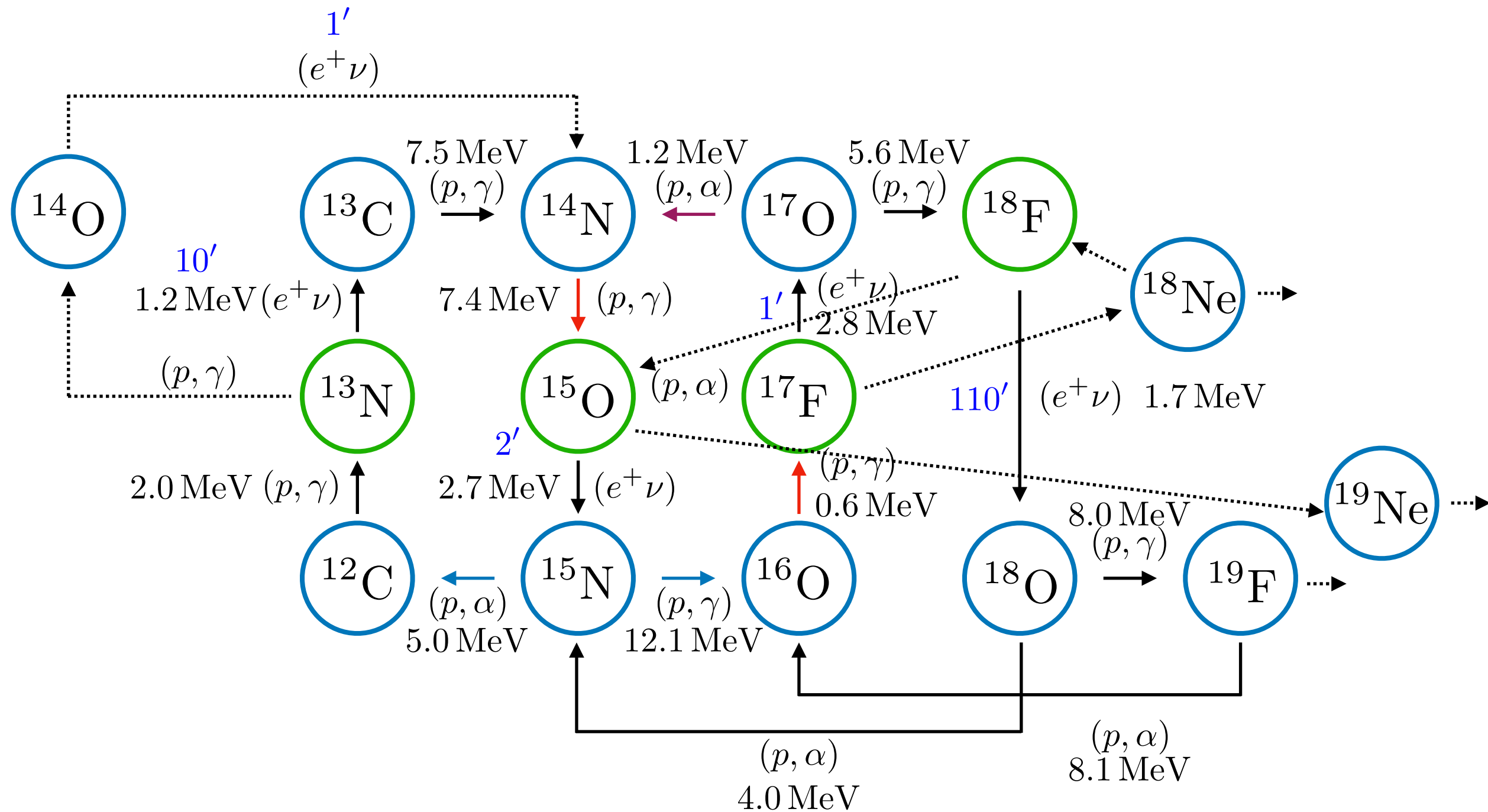
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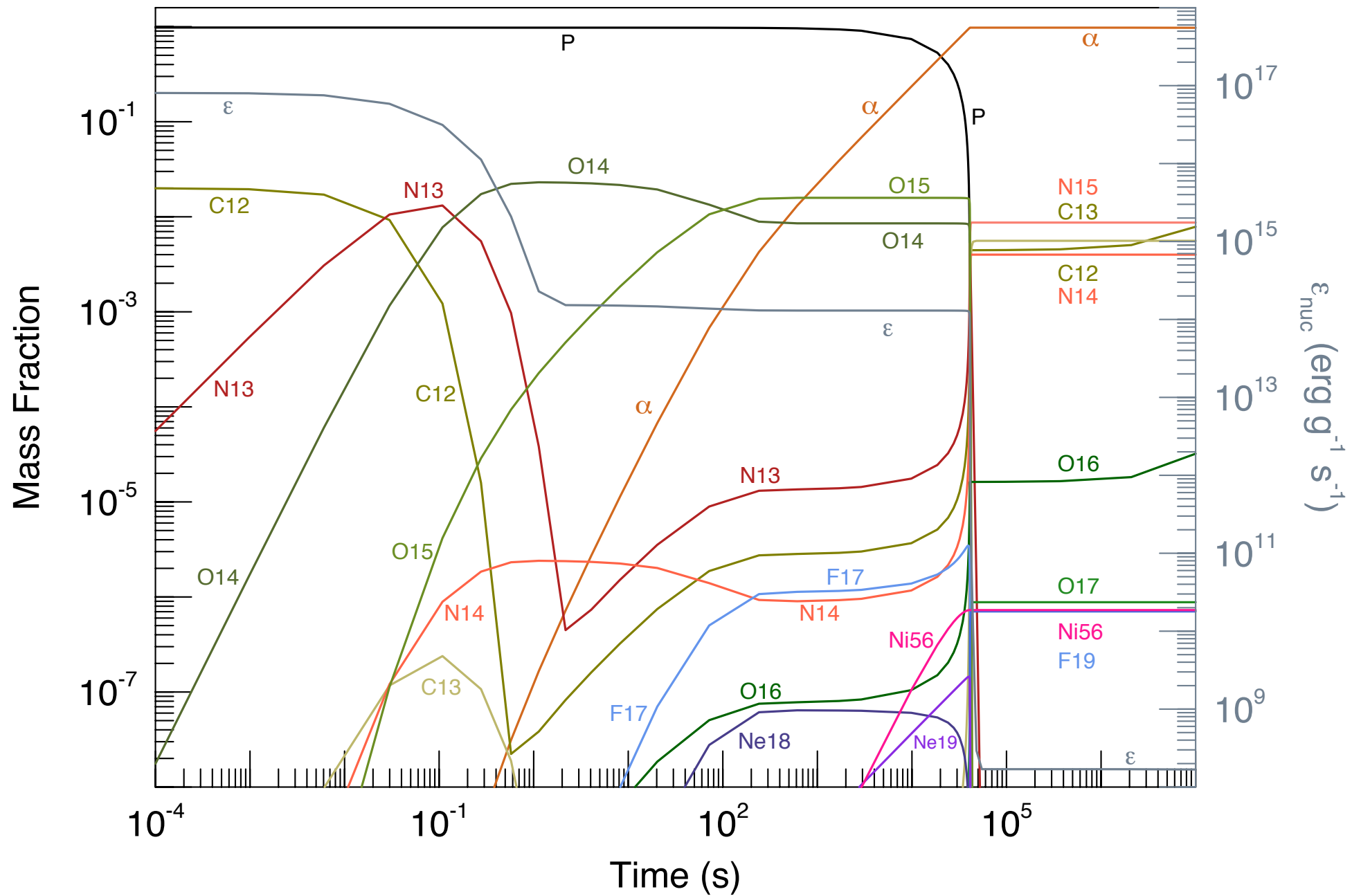
More CNO branches. HCNO and breakout

Is there a limit to how fast the CNO cycles can run? At high temperatures, some reactions become faster than beta decays



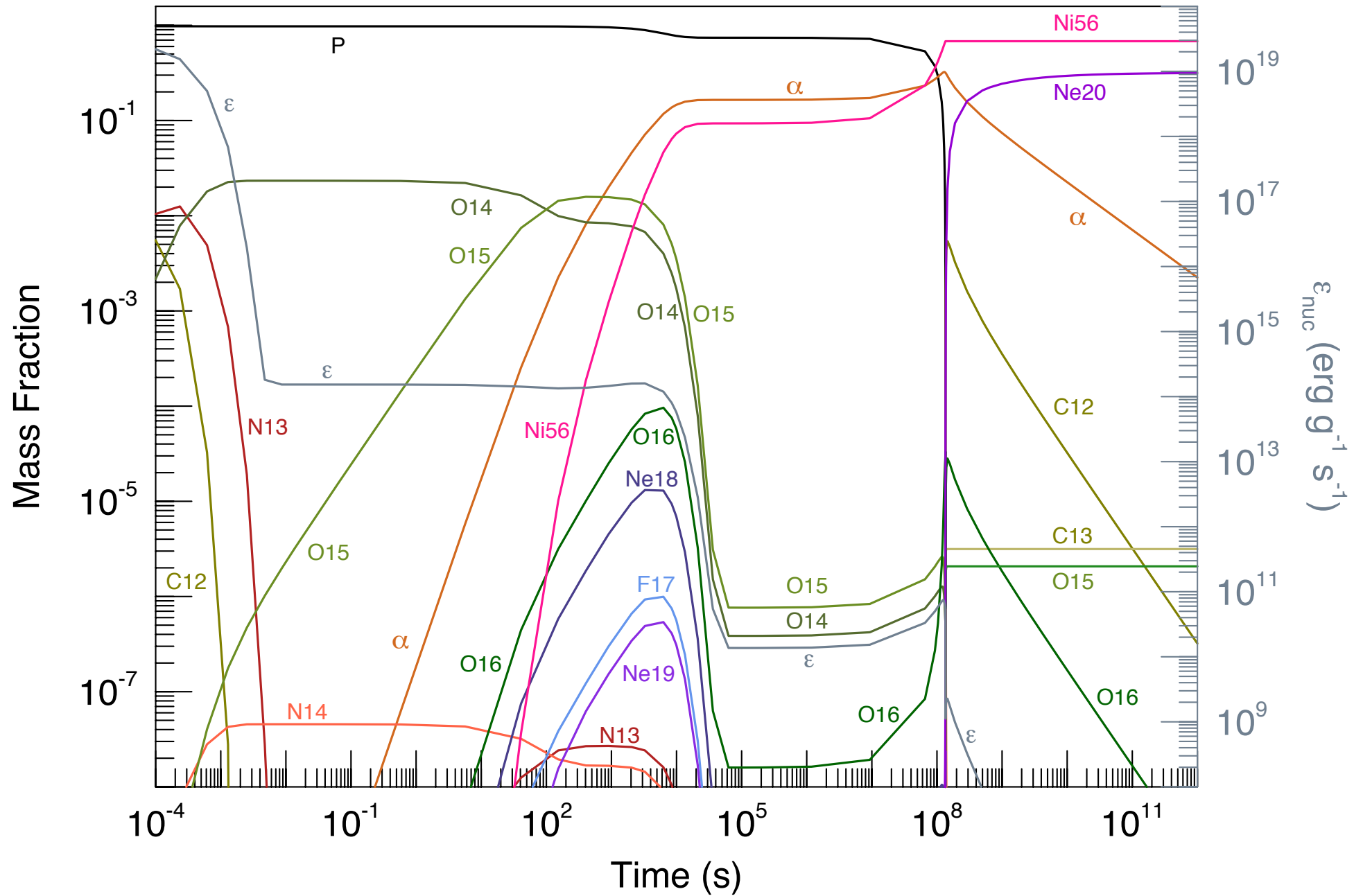
More CNO branches. HCNO and breakout

Beta Limited CNO cycles, hydrostatic burn, $T=300 \times 10^6 \text{ K}$ $\rho=150 \text{ g cm}^{-3}$



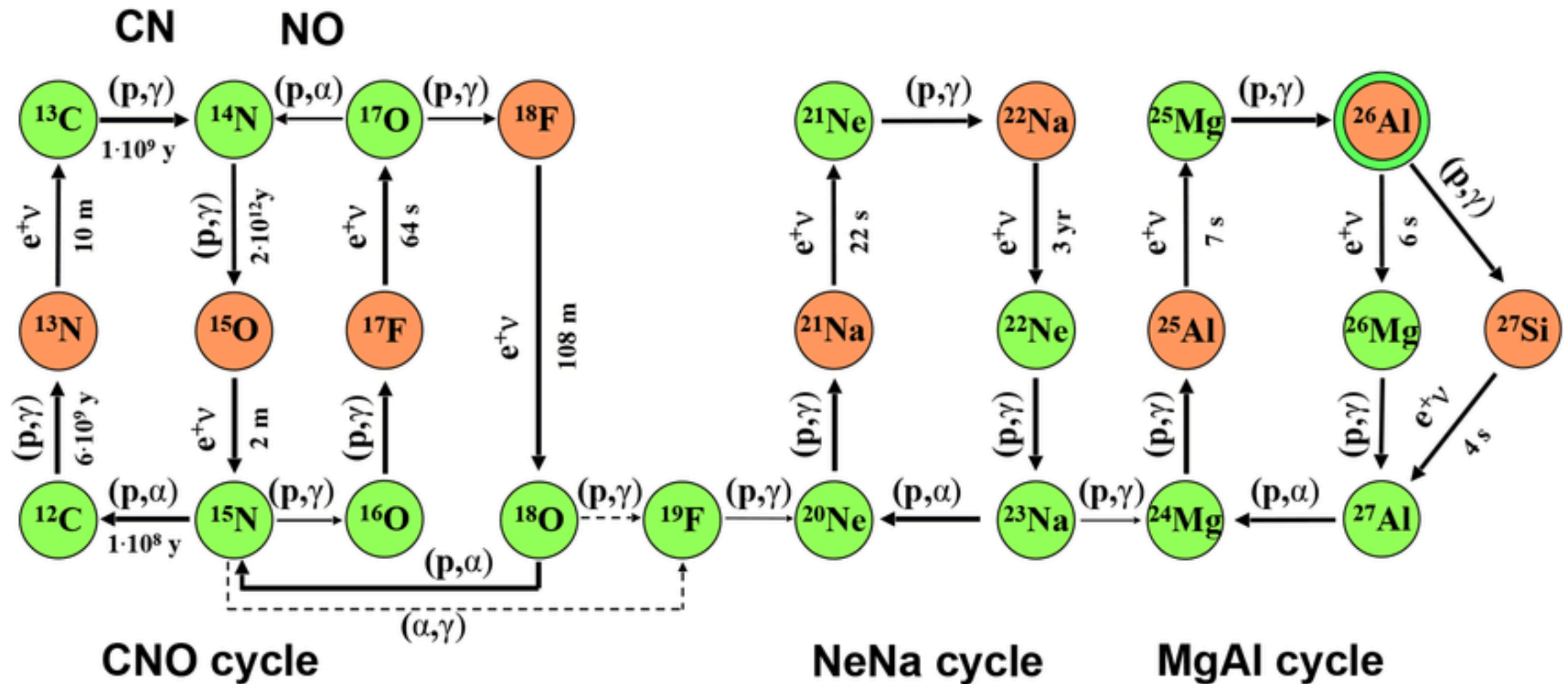
More CNO branches. HCNO and breakout

CNO rp breakout, hydrostatic burn, $T=600 \times 10^6$ K $\rho=150$ g cm $^{-3}$



High temperature hydrogen burning

More possibilities open up at high temperatures. Two important reaction chains for hydrogen burning are the NeNa and MgAl cycles

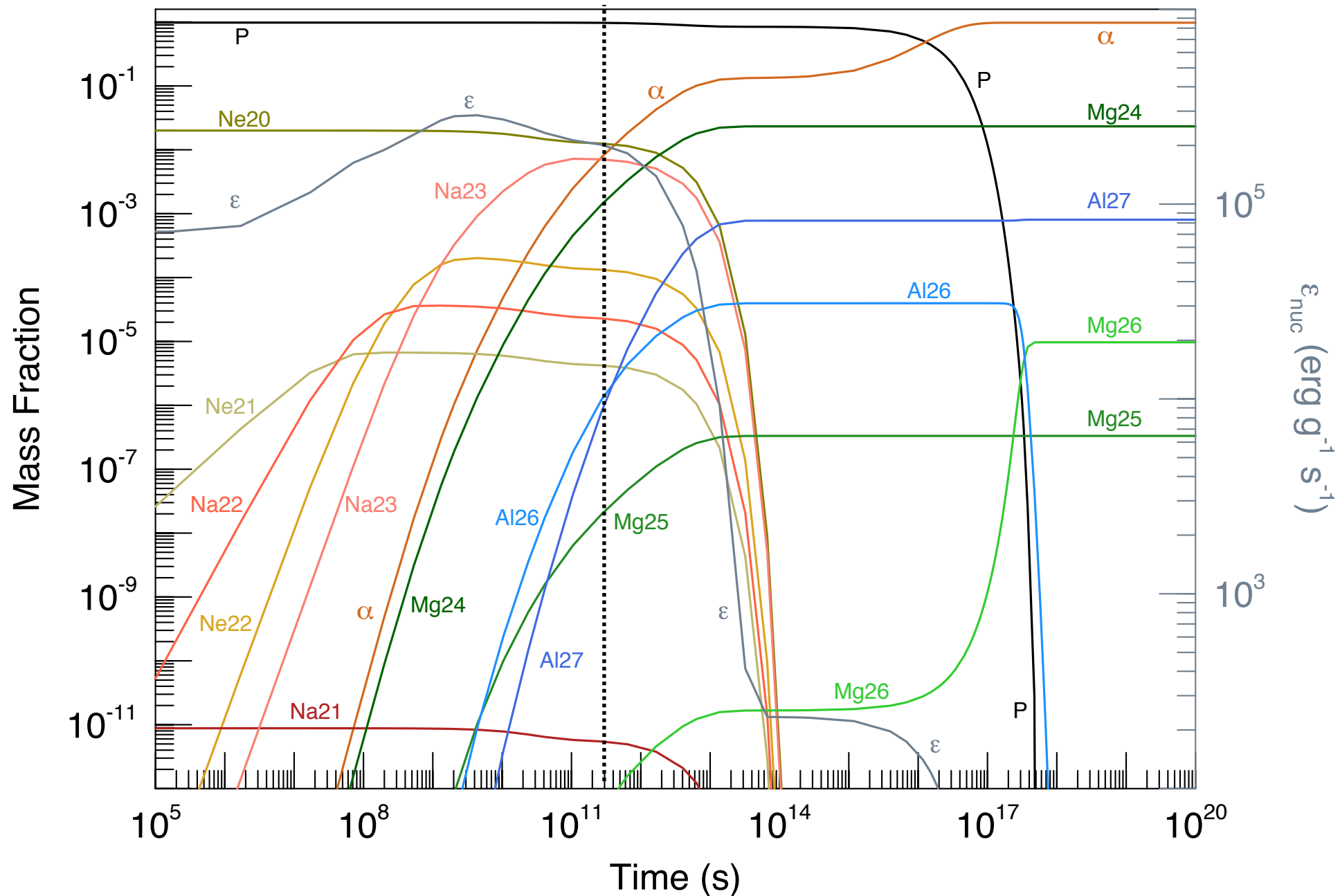


High temperature hydrogen burning

Example for $T=50$ million K. The most important product is Mg24 which reaches equilibrium after ~ 10000 yr. Aluminium is also strongly produced, first as Al26 and then as Al27.

Al26 is radioactive with a half life of 7×10^5 yr

Ne-Na and Mg-Al cycles, hydrostatic burn, $T=50 \times 10^6$ K $\rho=100$ g cm $^{-3}$

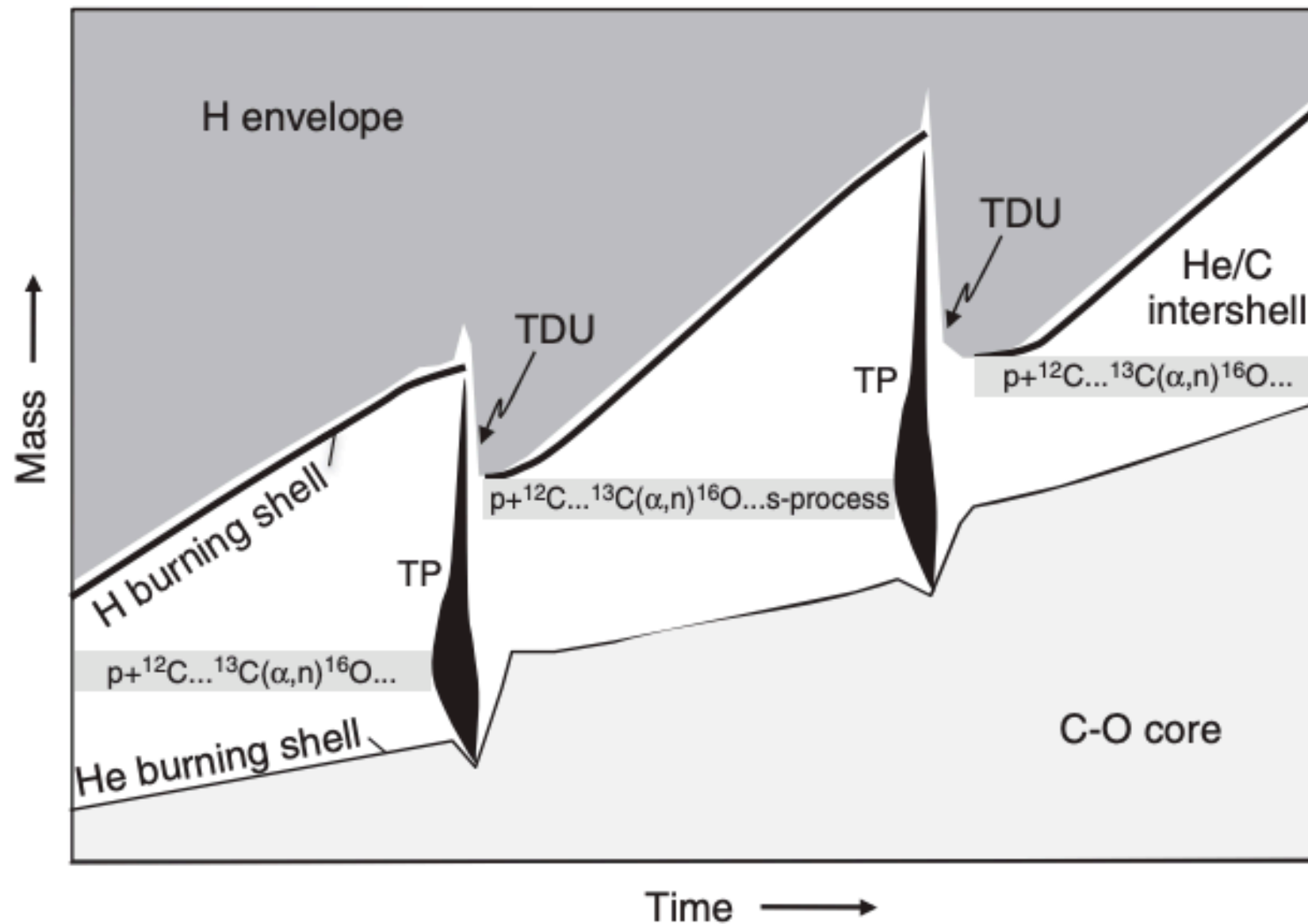


High temperature hydrogen burning

High temperatures possible in special astrophysical environments!

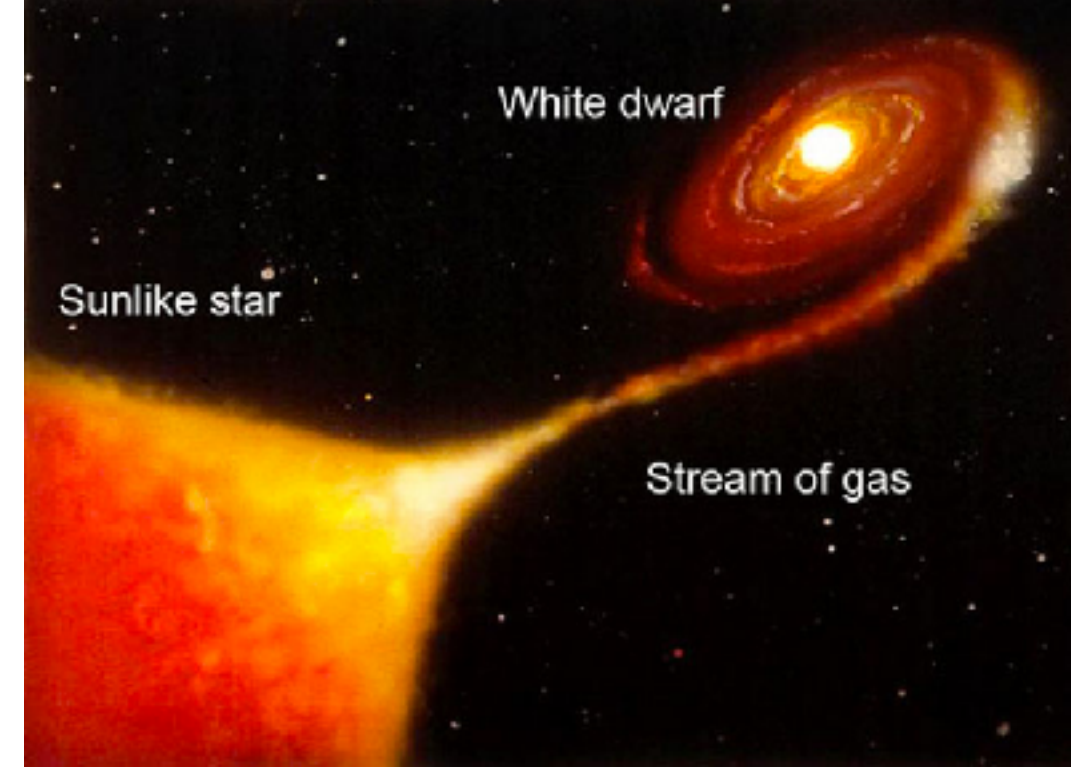
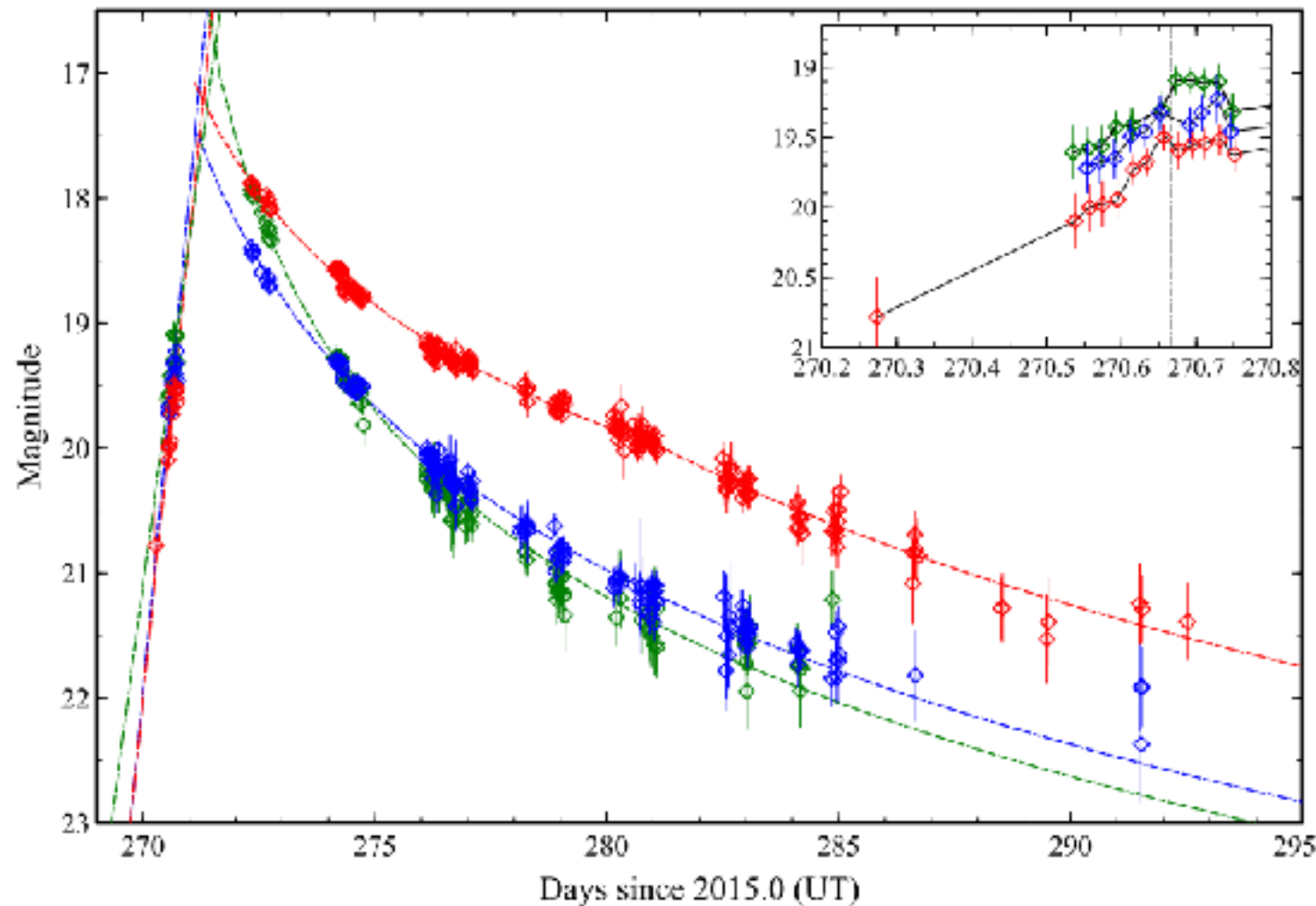
Cold CNO: $T_9 < 0.08$, Hot CNO and NeNa: $T_9 \sim 0.08 - 0.3$, breakout: $T_9 > 0.3$

Hot-bottom burning on the AGB



Nova eruptions

Nova explosions on accreting white dwarfs ($T_9 = 0.4$)



A WD accretes hydrogen-rich material from a companion star at slow rate (ca: $10^{-9} M_{\text{sun}}/\text{yr}$).

Runaway ignition occurs once the matter piles up and becomes dense (partly degenerate) and hot with $T > 10^7$ K
Nuclear burning can continue for up to a few weeks. After that the temperature drops.

All H-burning cycles can operate but for a limited time.

An earth-mass of material or so is ejected into the ISM: ^{15}O , ^{17}O , ^{22}Na , ^{26}Al

For H- accretion, models suggest that more material is ejected than accreted \rightarrow WD mass is shrinking!

Nova eruptions

Detection of radioactive ${}^7\text{Be}$ in a nova explosion!


Novae may be important production sites for Lithium

nature
International journal of science

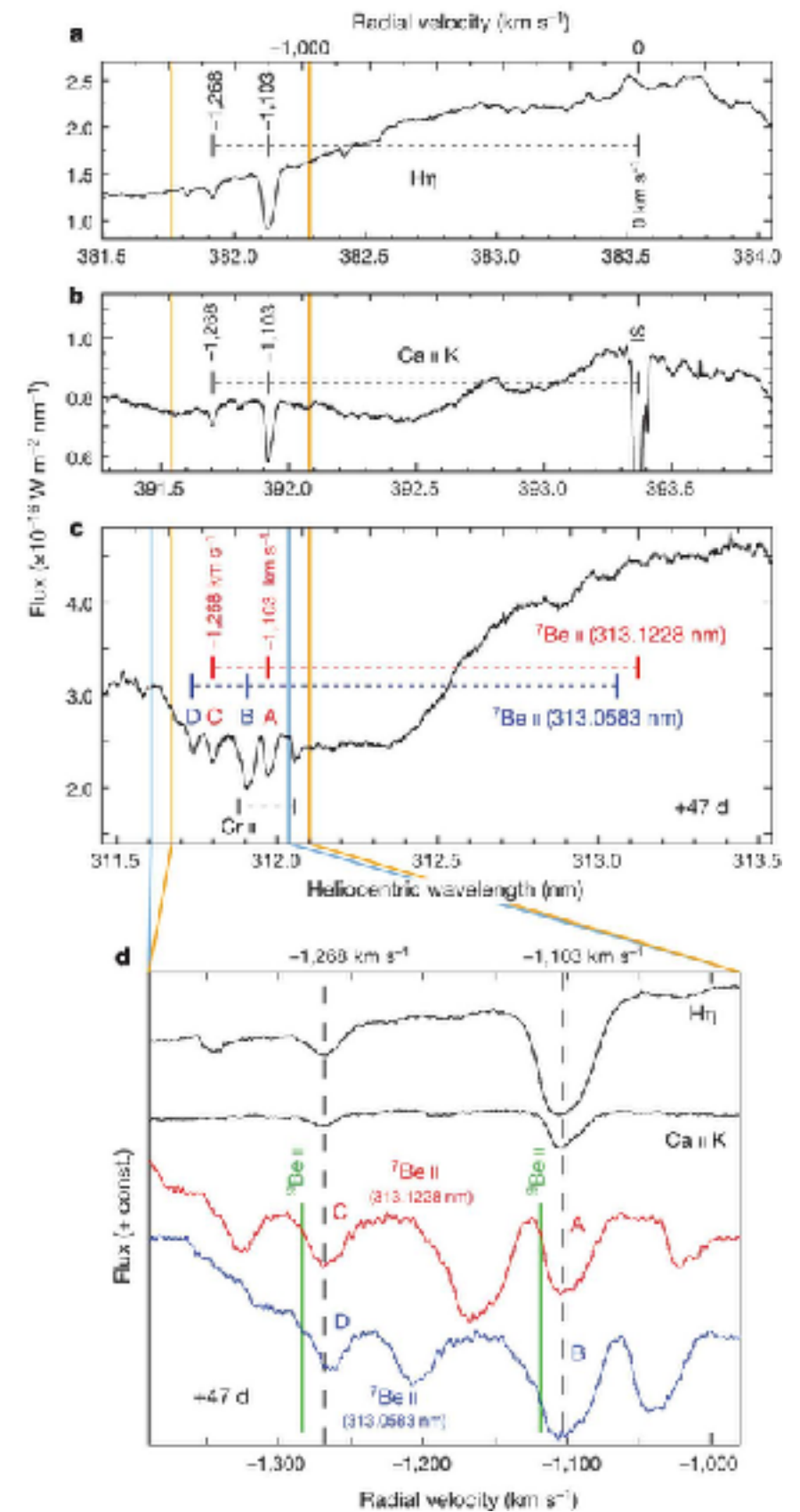
Letter | Published: 18 February 2015

Explosive lithium production in the classical nova V339 Del (Nova Delphini 2013)

Akito Tajitsu , Kozo Sadakane, Hiroyuki Naito, Akira Arai & Wako Aoki

Nature **518**, 381–384 (19 February 2015) | [Download Citation](#) 

Type I X-ray bursts



Type I X-ray bursts

A similar situation can arise when the accretion is a neutron star

Typical bursts last for 10-100s

Recurrence: hours-days

Superbursts: duration ~hour, recurrence every few years

Burning can be H/He or C depending on accretion rate

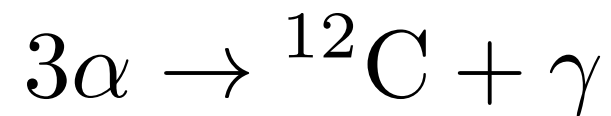
Much higher temperatures than classical Novae

~Zero matter ejected due to high NS gravity. ISM not enriched

Beyond H-burning: Helium at $T_8 > 1$

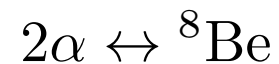
An important problem with the formation of elements heavier than He: **No stable nuclei at $A=5$ and $A=8$!**

How can one explain the existence of Carbon?



The reaction proceeds in two steps:

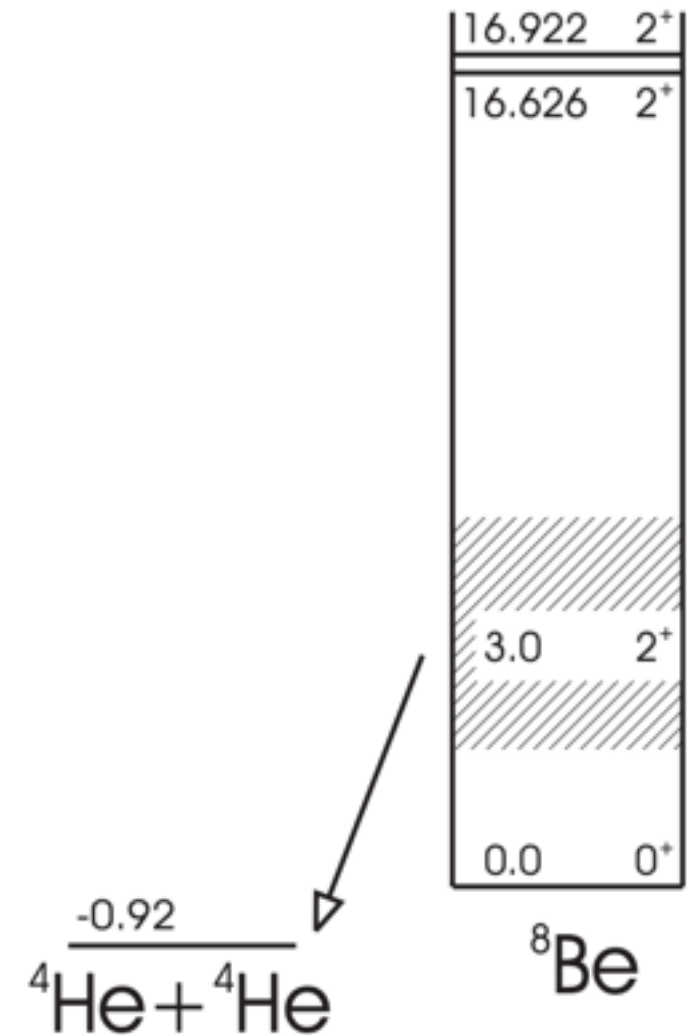
First 2 alpha particles fuse to ${}^8\text{Be}$.



This product is unstable, but only by 92 keV.

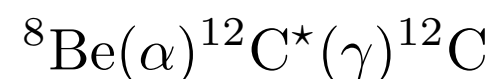
The ground state lifetime is $\sim 10^{-16}$ s, which is longer than the timescale of a non-resonant scatter

This means that both forward and backward reactions take place and a small equilibrium amount of ${}^8\text{Be}$ is build up



Once ${}^8\text{Be}$ is in equilibrium it can capture another α particle to form ${}^{12}\text{C}$.

This reaction is also dominated by a resonance at stellar energies, first predicted by Hoyle



Beyond H-burning: Helium at $T_8 > 1$

Next: $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$

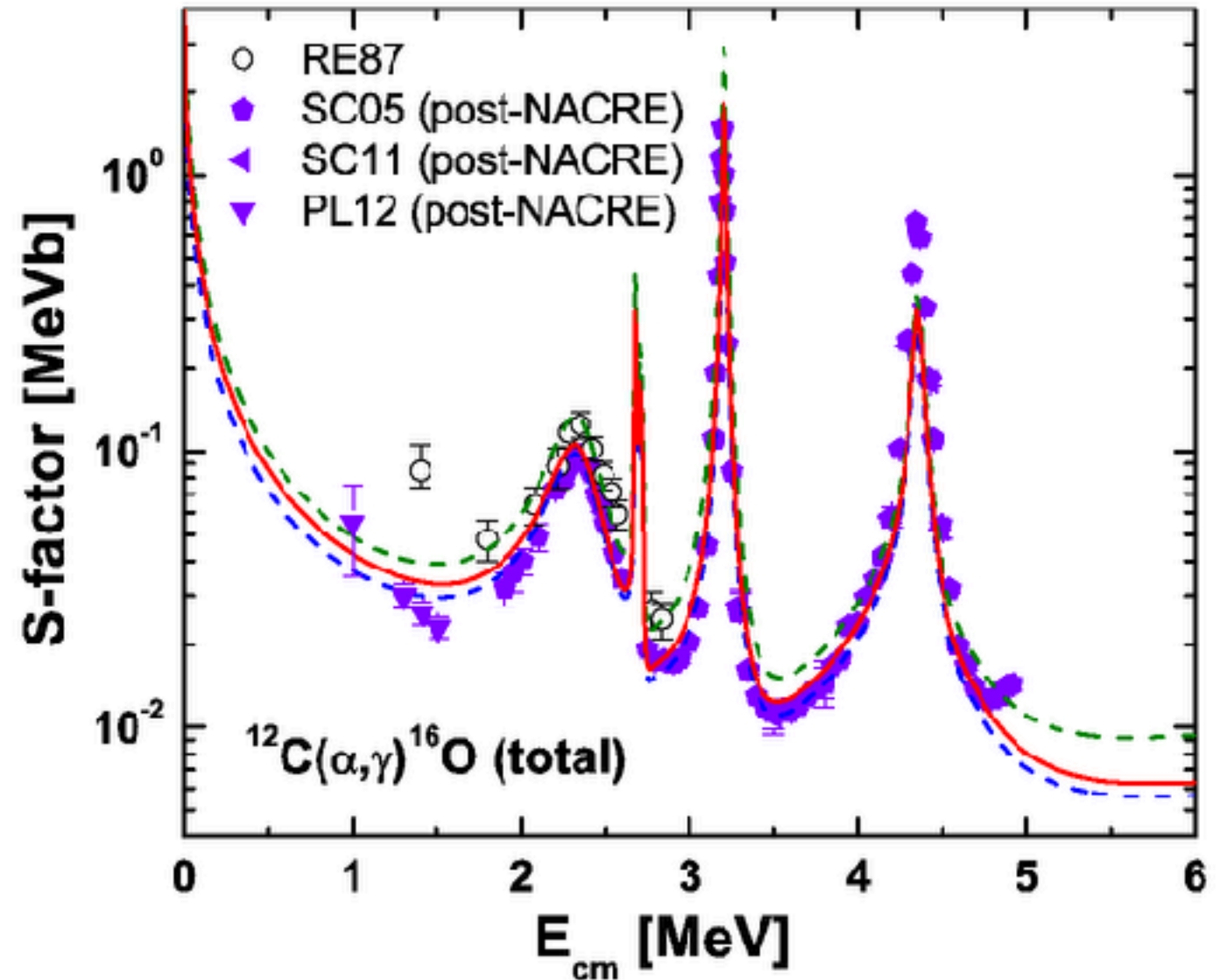
The reaction rate for a long time was highly uncertain, due to many resonances at low energies. The situation has somewhat improved in the past few years

Other reactions

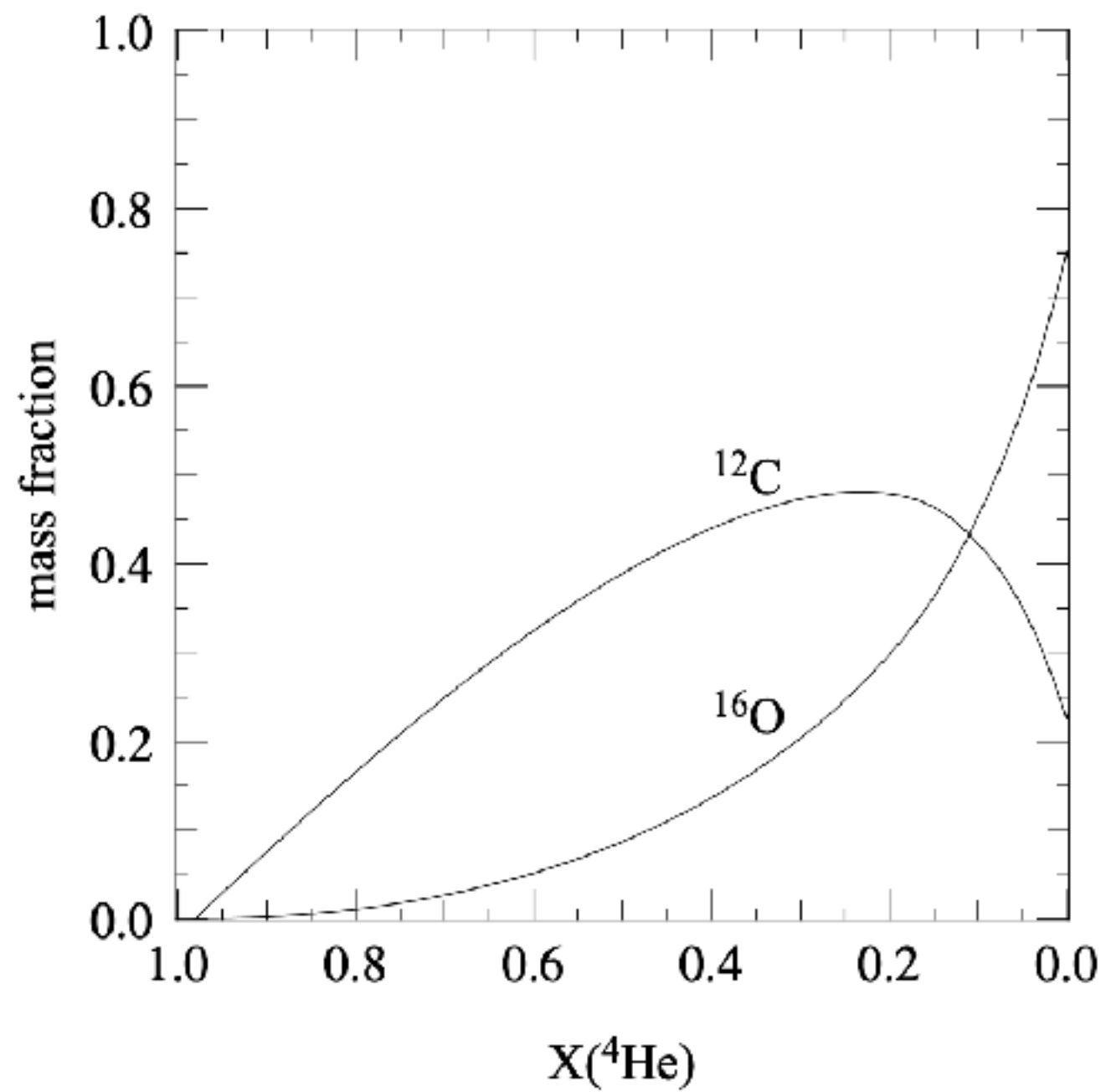
$^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$ less important

$^{14}\text{N}(2\alpha, 2\gamma)^{22}\text{Ne}$ occurs at low T_8

$^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$ neutron source!

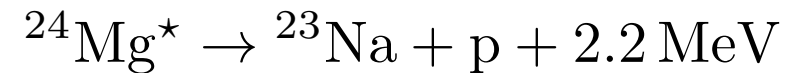
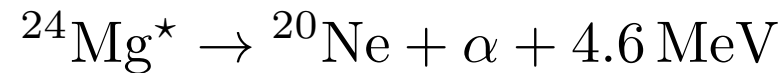
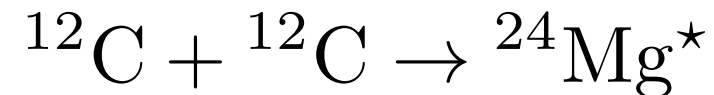


Beyond H-burning: Helium at $T_8 > 1$



Advanced burning

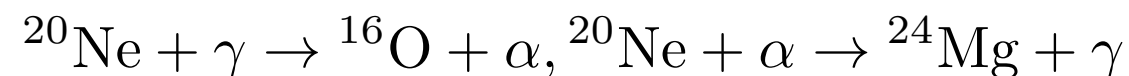
Carbon burning: Initially no light elements are present. The main reaction is:



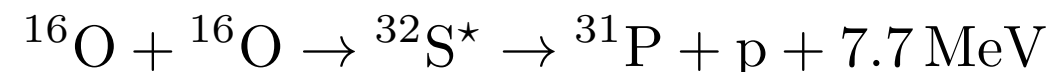
These “exit channels” have roughly the same probability. The light particles emitted are easily captured by other particles $^{23}\text{Na}(\text{p}, \alpha)^{20}\text{Ne}$, $^{20}\text{Ne}(\alpha, \gamma)^{24}\text{Mg} \dots$. The final composition is a mixture of O, Ne, Mg, Na. ^{23}Na is important as it can participate in urca cooling

Neon and Oxygen burning

Oxygen is a doubly magic nucleus and consequently more stable than Neon. Neon burning initiates at lower temperatures. Two main reactions, one of which is endothermic (photo-disintegration)



Oxygen burning proceeds via 2 main channels $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{32}\text{S}^* \rightarrow ^{28}\text{Si} + \alpha + 9.6 \text{ MeV}$



The light particles are again immediately captured allowing for many side reactions

Advanced burning

Silicon nuclei cannot fuse due to the extremely large Coulomb barrier. Instead, Si is transformed via a series of photo-disintegrations and alpha captures. $T^9 > 9$ leads to nuclear statistical equilibrium.

Mostly Ni56 and a healthy mixture of iron-group elements. Ni decays to iron with a half-life of 6 days

