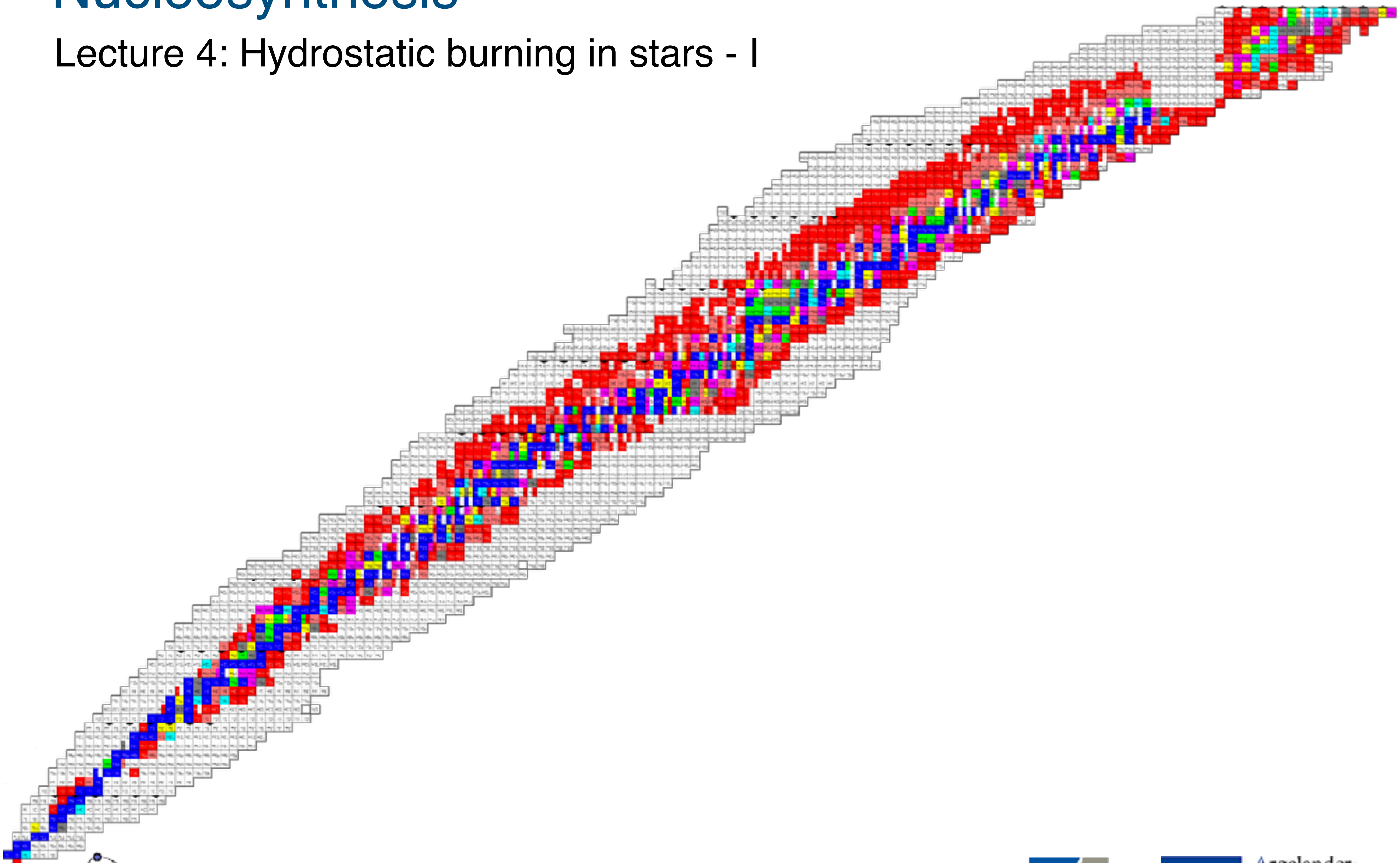
Nucleosynthesis

Lecture 4: Hydrostatic burning in stars - I



Bonn-Cologne Graduate School of Physics and Astronomy

UNIVERSITÄT BONN

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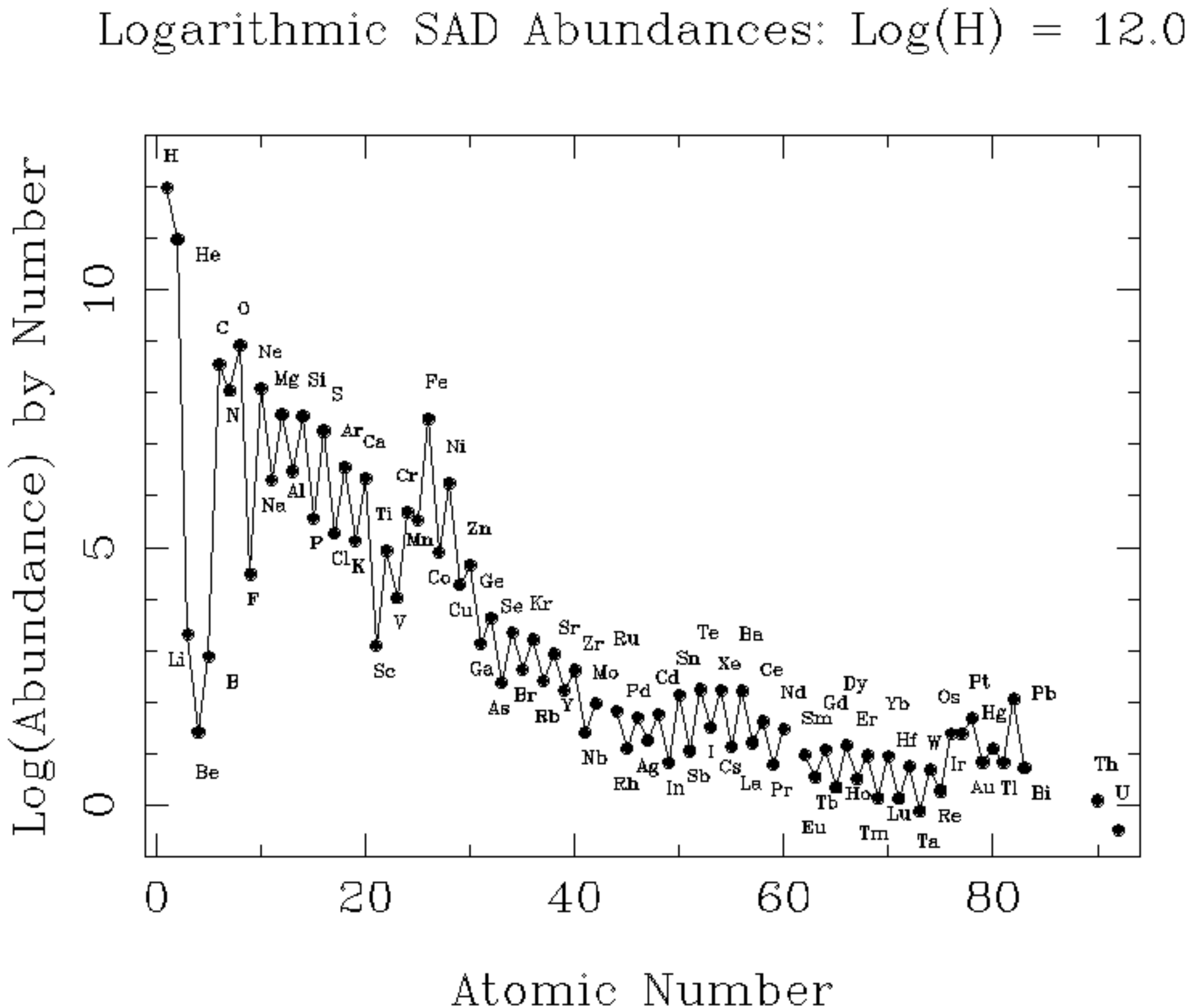
Overview

- **Lecture 1:** Introduction & overview **April 18**
- **Lecture 2:** Thermonuclear reactions **April 25**
- **Lecture 3:** Big-bang nucleosynthesis **May 2**
- **Lecture 4:** Thermonuclear reactions inside stars — I (H-burning) **May 7**
- **Lecture 5:** Thermonuclear reactions inside stars — II (advanced burning) **May 17**
- **Lecture 6:** Neutron-capture and supernovae — I **May 23**
- **Lecture 7:** Neutron-capture and supernovae — II **June 6**
- **Lecture 8:** Thermonuclear supernovae **June 27**
- **Lecture 9:** Li, Be and B **July 4**
- **Lecture 10:** Galactic chemical evolution and relation to astrobiology **July 11**

Paper presentations I **June 21**

Paper presentations II **June 28**

Overview of previous lectures



Overview of previous lectures

To evaluate the rates of nuclear reactions, and the composition evolution, one needs to know the plasma conditions (T, ρ, X^i's) and the nuclear cross sections as a function of energy

$$r_{a,X} = (1 + \delta_{Xa}) N_X N_a \langle \sigma v \rangle$$

For non-resonant reactions when experimental data are available, <σv> can be evaluated empirically (S-factor, approximations etc)

Complications when:

1. Non-resonant cross sections are too small to measure in the lab (e.g. most weak reactions

2. Resonances exist in the range of effective stellar energies

Composition change

$$\frac{dX_i}{dt} = A_i \frac{m_u}{\rho} \left(- \sum r_{\text{reactions that destroy } i} + \sum r_{\text{reactions that create } i} \right)$$

Overview of previous lectures

Application in the primordial Universe

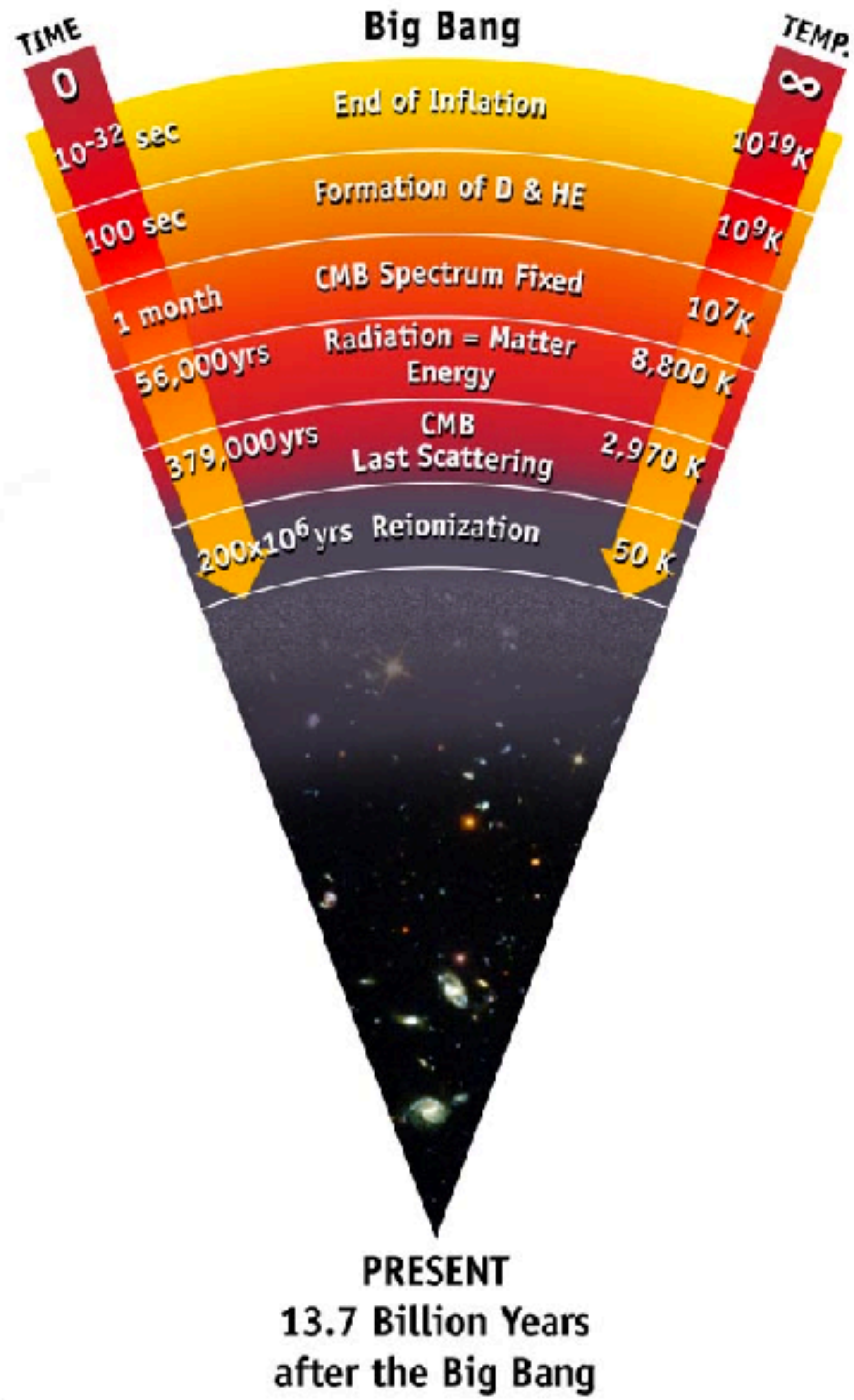
After inferring the conditions and kinematics using homogeneous cosmology, we saw that BBN can account for the abundances of most H and He isotopes

Need for further nucleosynthesis

The Universe created in the BB is incomplete. Further nucleosynthesis needed to create what we see today

Nucleosynthesis in stars

Just like in BBN, we need to infer the plasma conditions, this time using stelar evolution theory



Overview of stellar properties

The Sun is the most well studied star

$$R_\odot \simeq 695,000\,\mathrm{km}$$

$$M_\odot \simeq 2 \times 10^{30}\,\mathrm{kg}$$

$$L_\odot \simeq 4 \times 10^{26}\,\mathrm{W}$$

$$X, Y, Z \simeq 0.78, 0.24, 0.02$$

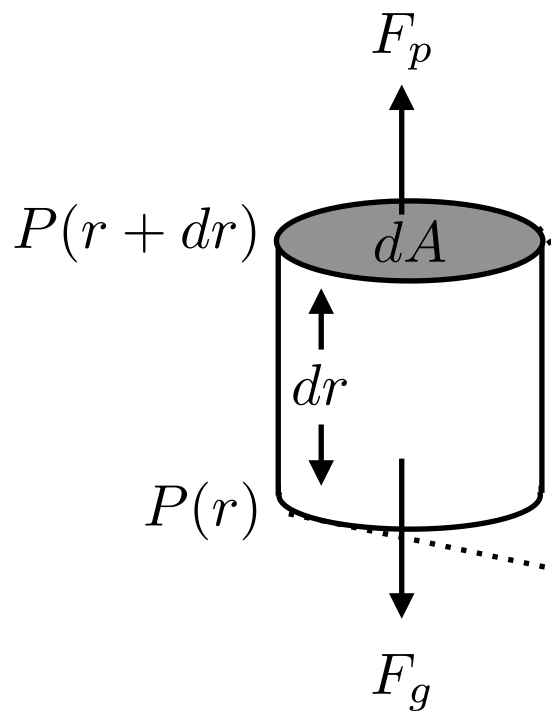
How long would it take for a surface element to reach the centre of the Sun?

$$\ddot{r} = -\frac{Gm}{r^2} \rightarrow |\ddot{r}| = \frac{R}{\tau_{\mathrm{ff}}^2} \simeq -\frac{GM_\odot}{R_\odot^2} \Rightarrow \tau_{\mathrm{ff}} \simeq \sqrt{\frac{R_\odot^3}{GM_\odot}} \sim 0.5\,h$$

The Sun and the stars remain stable for much longer than that, which means that the internal pressure balances exactly the pull of gravity —> **Hydrostatic Equilibrium**

Stellar structure*

Hydrostatic Equilibrium



$$F_{\text{pressure}} = -F_{\text{gravity}}$$

$$dPdA = -GM(r)m/r^2$$

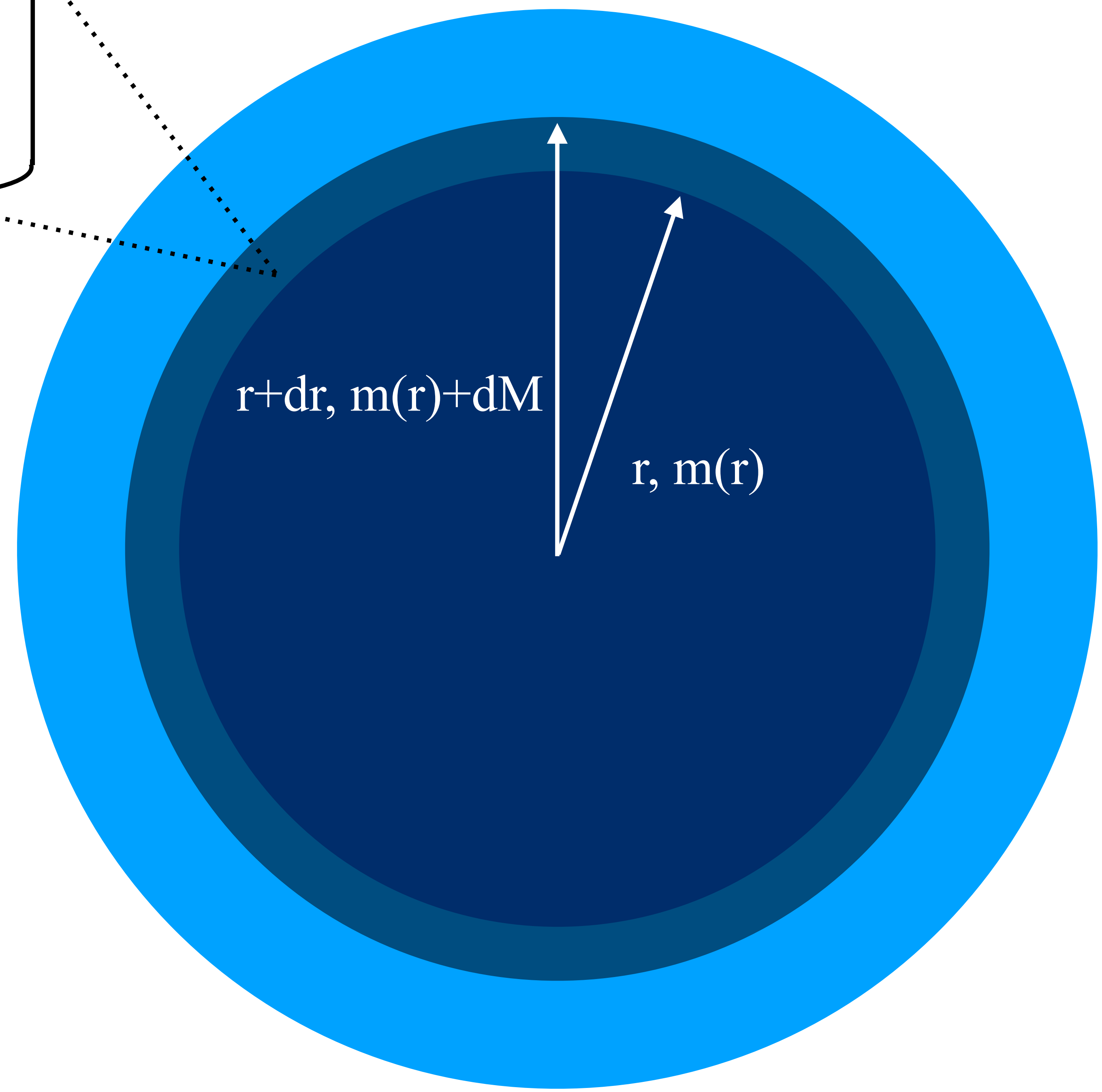
$$dPdA = -GM(r)\rho(r)dAdr/r^2$$

$$\frac{dP}{dr} = -GM(r)\rho(r)/r^2$$

Usually, we use the mass, m instead of the radius r as a free parameter. The mass contained inside a spherical shell between $r, r+dr$ is $dM = 4\pi\rho r^2 dr$, therefore:

$$\frac{dP}{dM} = -\frac{GM}{4\pi r^4}$$

More info needed to solve for the structure



*we will only study spherically symmetric stars (in 1D)

Stellar structure and evolution equations

$$\frac{\partial P}{\partial M} = -\frac{GM(r)}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}$$

Equation of motion

$$P = P(\rho, T, X_i)$$

Equation of state

$$\frac{\partial r}{\partial M} = \frac{1}{4\pi r^2 \rho}$$

Conservation of mass

$$\frac{\partial l}{\partial M} = \epsilon_{\rm nuc} - \epsilon_{{\rm nuc},\nu} - \epsilon_\nu - T\frac{\partial s}{\partial dt}$$

Conservation of energy

$$\frac{\partial T}{\partial M} = -\frac{GM}{4\pi r^4}\frac{T}{P}\nabla$$

$$\nabla = \nabla_{\rm rad} = \frac{3\kappa}{16\pi ac}\frac{lP}{MT^4}, \nabla_{\rm rad} \le \nabla_{\rm ad}$$

$$\nabla = \nabla_{\rm ad} + \Delta\nabla, \nabla_{\rm rad} > \nabla_{\rm ad}$$

Energy transport

$$\kappa = \kappa(\rho, T, X_i)$$

Opacity

$$\frac{\partial X_i}{\partial t} = \frac{A_i m_u}{\rho}\left(-\sum_{j,k} r_{ij,k} + \sum_{kl} r_{kl,i}\right) + [\text{mixing terms}]$$

Composition evolution

$$Q = Q(i,j,k) - Q(i,j,k)_\nu$$

Q-values

Overview of stellar evolution

Temperatures and luminosities of stars vary greatly Nevertheless, they are occupy specific regions of the H-R diagram.

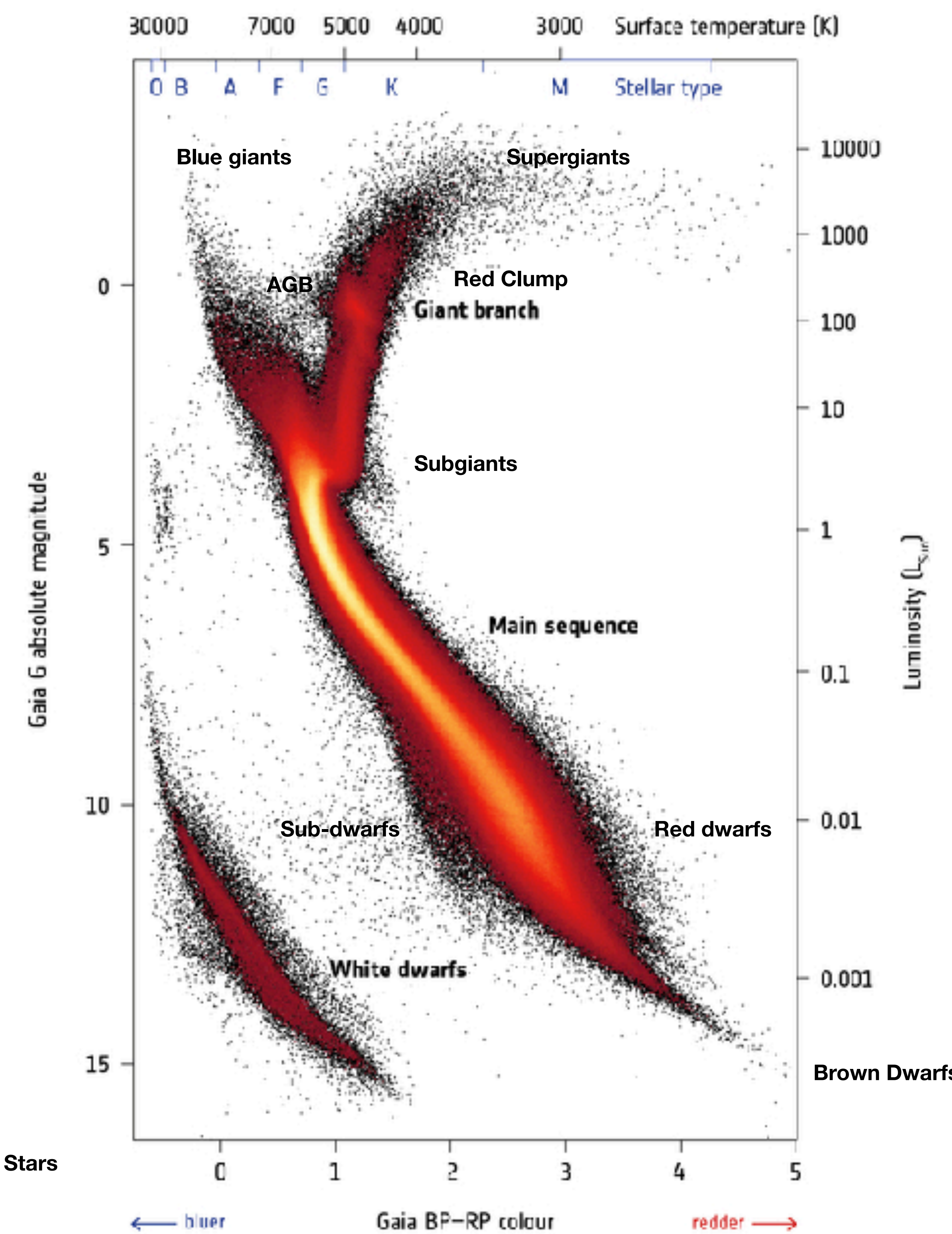
Explaining the H-R properties and the relative populations of stars at each region has been one of the great successes of stellar evolution theory

Stellar luminosity and lifetime depend strongly on mass

Mass	Star											
0.013–0.08	Brown dwarf	D-C										
0.08–0.4	Red dwarf	H-C [MS]										He WD
0.4–2 (1.5)	Low mass star	H-C [MS] pp CNO	H-S [RGB]	1. DU	HeF	He-C H-S [HB,RC]	He-S H-S [AGB]			3. DU	PNN	CO WD
2–4	Inter-mediate mass star	H-C [MS]	H-S [RGB]	1. DU		He-C H-S	He-S H-S [AGB]			3. DU	PNN	CO WD
4–9		H-C [MS]	H-S [RGB]	1. DU		He-C H-S	2. DU He-S H-S [AGB]			3. DU	PNN	CO WD
9–11		H-C [MS]	H-S [RGB]	1. DU		He-C H-S	He-S	C-C He-S [SAGB]	2. DU He-S H-S	3. DU	PNN	ONe WD
11–100	Massive star	H-C [MS]	He-C H-S	C-C He-S ...	Ne-C C-S ...	O-C Ne-S ...	Si-C O-S ...	CC SN II/Ib/Ic				BH or NS

Table from ILIADIS

➔ GAIA'S HERTZSPRUNG-RUSSELL DIAGRAM



Neutron Stars

Overview of stellar evolution

Pre-main sequence evolution

- Initially the gas is in free fall
- As temperature and pressure increase, hydrostatic equilibrium is achieved
- Deuterium burning temporarily halts contraction and allows for further mass accretion at the surface
- Luminosity is carried by convection, thus the material becomes fairly well mixed

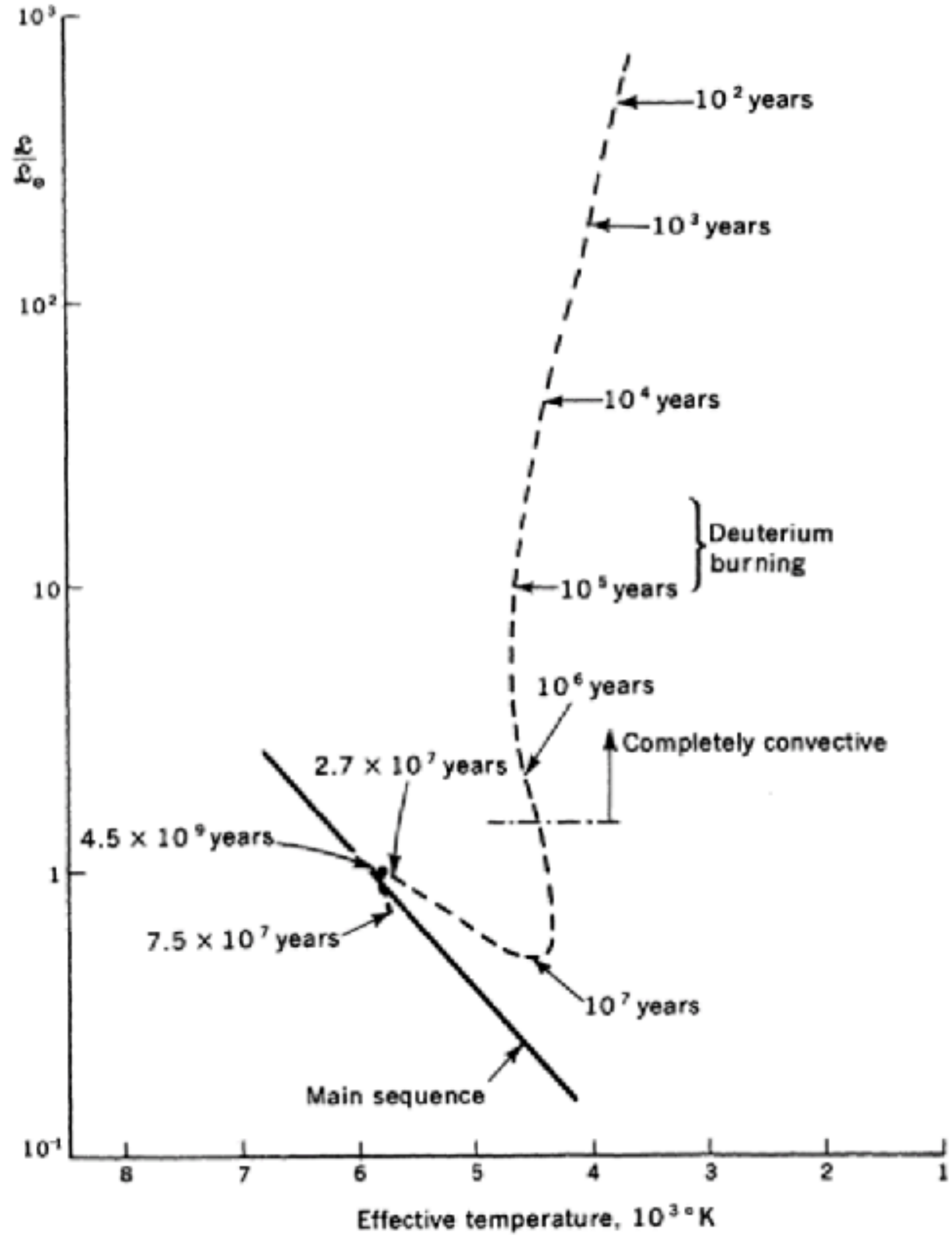


Figure from CLAYTON

Overview of stellar evolution

Brown dwarfs $0.013\,\mathrm{M}_\odot \leq M \leq 0.08\,\mathrm{M}_\odot$

- Should be very abundant in the Galaxy. Because they are faint, they may contribute to baryonic dark matter
- Central temperature too low for hydrogen burning. Only deuterium burning occurs, contributing a small amount of energy
- Lithium burning does not occur
- The core is degenerate

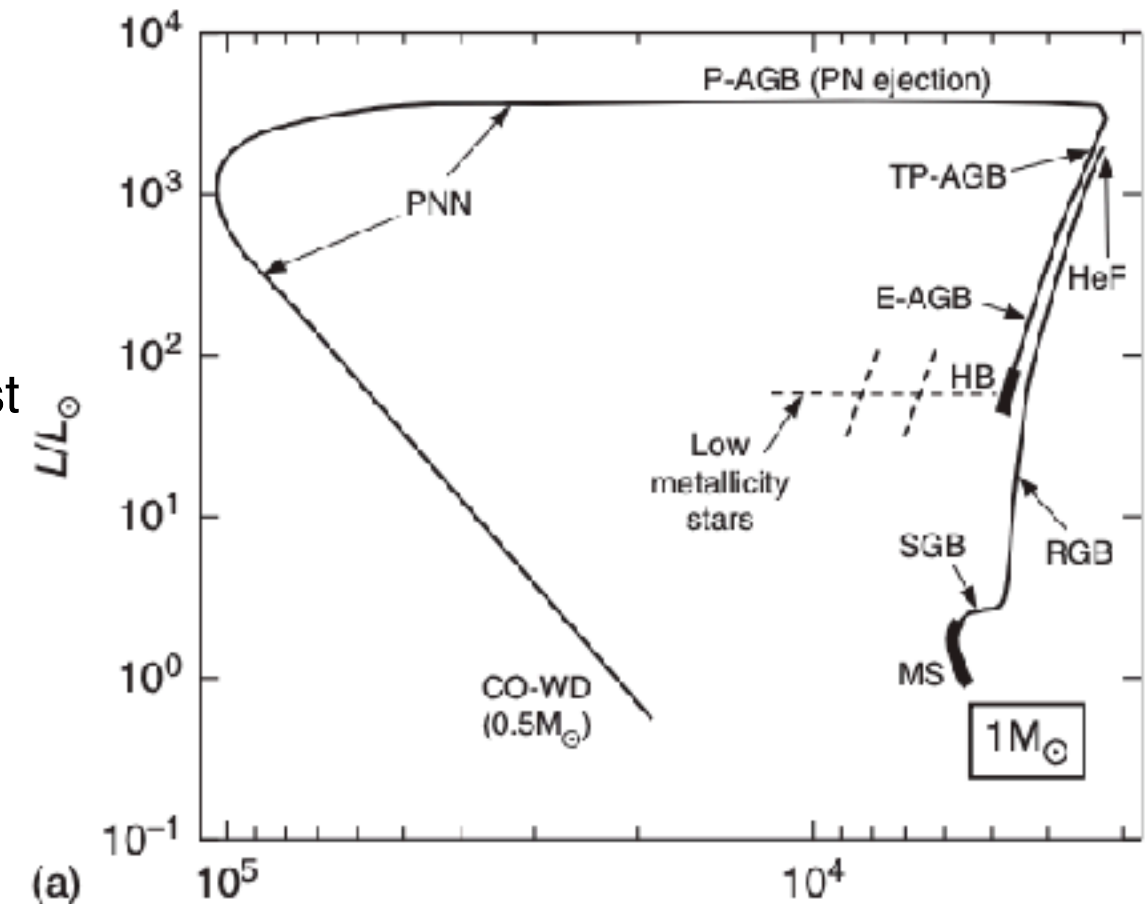
Red dwarfs $0.08\,\mathrm{M}_\odot \leq M \leq 0.4\,\mathrm{M}_\odot$

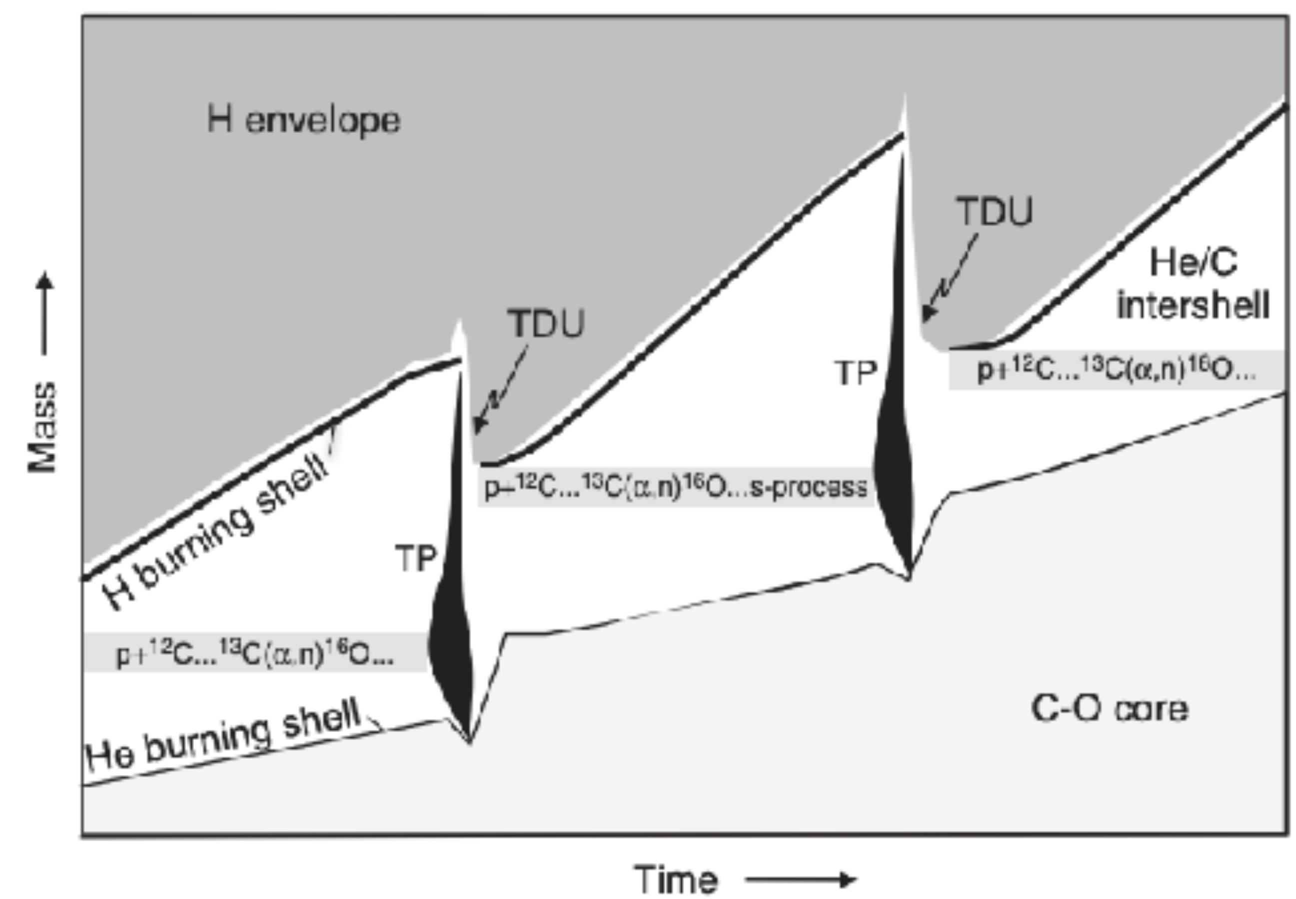
- Sufficiently high mass for hydrogen burning
- Their main-sequence lifetime can be several thousand times the age of the Universe (practically immortal)
- Not enough mass for helium fusion; eventually, they will become helium white dwarfs

Overview of stellar evolution

low-mass stars $0.4\,\mathrm{M}_\odot \leq M \leq 2\,\mathrm{M}_\odot$

- Below ~1.5 solar masses, core energy transport via radiation, above that, via convection
- After core H burning, the star is located at the bluest/hottest point on the MS
- H-burning continues in a shell, while the core contracts and the star leaves the MS
- Heating and shell burning eventually creates a fully convective envelope (SGB —> RGB)
- The core becomes degenerate, and when the temperature rises, a runaway is initiated (HeF)
- Eventually core He is exhausted and the core contracts again. An unstable double shell structure develops on the E-AGB
- Shell interaction leads to thermonuclear He runaway (TP-AGB)
- Finally the envelope is expelled and the star settles as a WD





Figures from ILIADIS

Overview of stellar evolution

Intermediate-mass $2\,\mathrm{M}_\odot \leq M \leq 11\,\mathrm{M}_\odot$

$M \leq 4\,\mathrm{M}_\odot$

- The helium core does not become degenerate on the RGB and therefore helium burning is stable. The subsequent evolution is very similar to lower-mass stars

$M \leq 8\,\mathrm{M}_\odot$

- These stars experience an additional dredge-up episode on the E-AGB. Hot-bottom burning creates convection and brings processed material to the surface

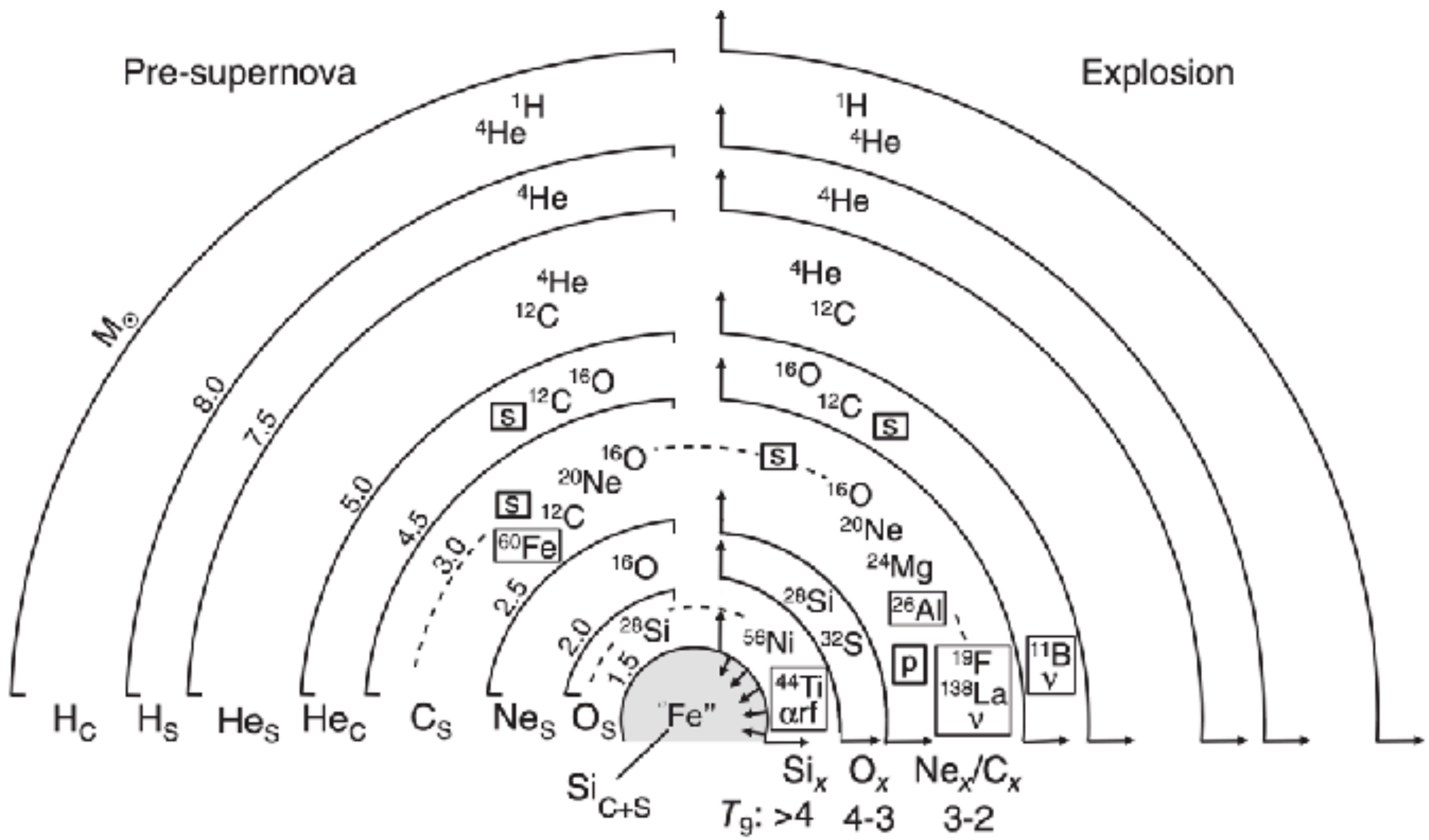
$M \leq 11\,\mathrm{M}_\odot$

- After core helium burning the core contracts. Heating due to contraction competes with neutrino losses (cooling). As a result, the maximum temperature is attained off-centre, and carbon ignites in a series of flashes under mildly degenerate conditions. The star becomes a super-AGB star and eventually settles as an ONeMg WD

Overview of stellar evolution

High-mass stars

Qualitatively very different evolution. Fusion continues to convert elements to heavier ones until a Fe core develops, surrounded by an onion-like structure.
The star collapses and creates a SN explosion



Figures from ILIADIS

Stellar Structure

What can we learn about a star without solving the detailed set of diff. equations?

The Virial Theorem

$$\frac{dP}{dM} = -\frac{GM(r)}{4\pi r^4} = -\frac{GM(r)}{r \times 3 \times (4/3)\pi r^3} \Rightarrow 3dPV(r) = -\frac{GM(r)dM}{r}$$

Integrating from the core to the surface of the star:

$$\int_c^s 3dPV(r) = -\int_c^s \frac{GM(r)dM}{r} \Rightarrow 3\int_c^s dPV(r) = E_{\text{grav}}$$

For the left hand side of the equation, integration by parts yields:

$$3\int_c^s dPV(r) = 3\left[PV\right]_c^s - 3\int_c^s PdV = -3\int_0^M \frac{P}{\rho}dM \Rightarrow \boxed{-E_{grav} = 3\int_0^M \frac{P}{\rho}dM}$$

This tells us that when the gravitational energy increases during contraction, the star must increase its internal pressure to maintain hydrostatic equilibrium

Stellar Structure

P/ρ **is also related to the internal energy per unit mass,** $P/\rho = u\zeta/3$

$$-E_{\rm grav} = \zeta E_{\rm int}, \zeta = 1, 2$$

Thus, for an ideal gas, contraction also leads to higher temperature

How does the pressure behave qualitatively?

$$E_{\rm grav} = -\int_0^M \frac{GM}{r} dM \sim \frac{G}{\langle r \rangle} M^2$$

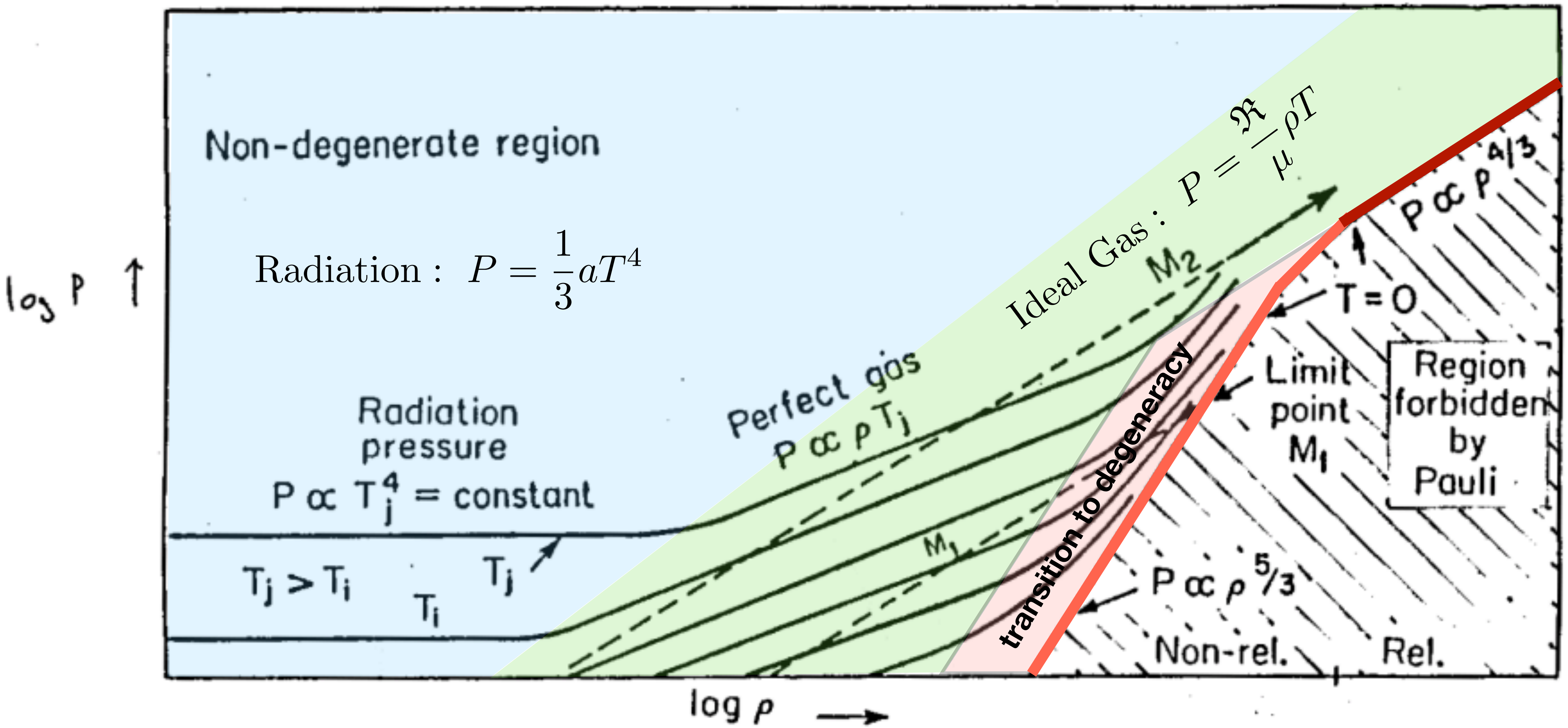
$$E_{\rm int} \propto \int_0^M P dV \sim \langle P \rangle V \sim \langle P \rangle \langle r \rangle^3$$

Therefore, from the Virial theorem it follows that, independently of the temperature and EoS:

$$\langle P \rangle \propto \frac{GM^2}{\langle r \rangle^4} \sim GM^{2/3} \langle \rho \rangle^{4/3}$$

Stellar Structure

Can we infer the temperature evolution of the core?
For that we also need some info about the EoS



Stars with masses above a certain threshold M_{ch}, can heat-up indefinitely

Stars below this limit will heat and then cool and become degenerate.
Hence, they reach a maximum temperature, which depends on the mass.
Since fusion of elements requires a certain min. temperature, a minimum mass also required

The Chandrasekhar mass

At high densities, pressure is provided only by the electron gas and does not depend on the temperature

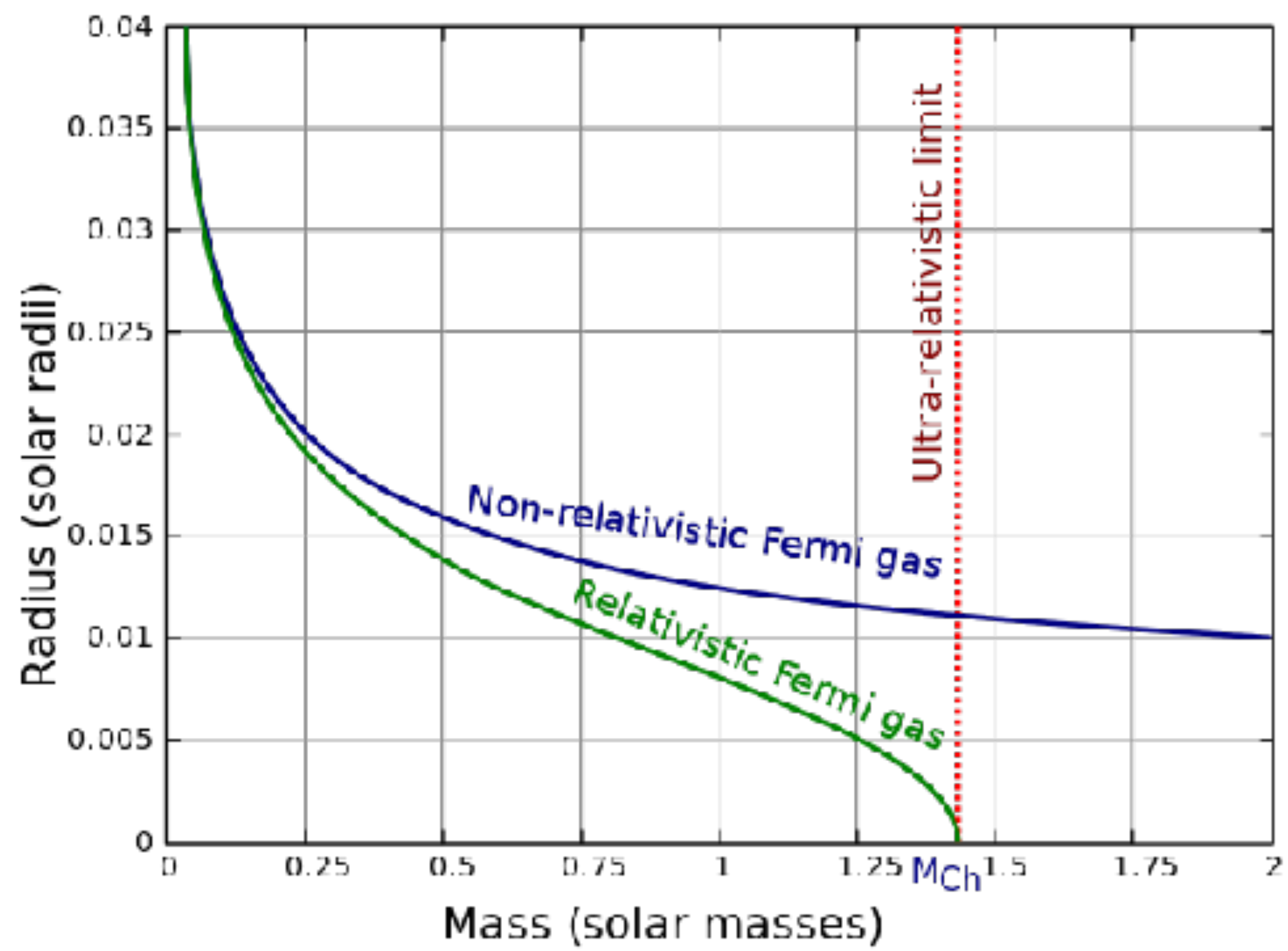
$$\text{non}-\text{relativistic degeneracy}: \quad P = K_1 \rho^{5/3}$$

$$\frac{dP}{dr} = -\frac{GM}{\rho^2} \rightarrow \langle P \rangle \propto \frac{M^2}{R^4}$$

$$\frac{M^2}{R^4} \propto \left(\frac{M}{R^3}\right)^{5/3} \rightarrow R \propto M^{1/3}$$

$$\text{relativistic degeneracy}: \quad P = K_1 \rho^{4/3}$$

$$\frac{M^2}{R^4} \propto \left(\frac{M}{R^3}\right)^{4/3} \rightarrow M = const.$$



$$M_{\rm ch} \simeq \frac{5.836}{\mu_e^2} \, \mathrm{M}_\odot, \;\; \mu_e := \left(\sum_i \frac{X_i Z_i}{A_i}\right)^{-1} \simeq 1.26 - 1.459 \, \mathrm{M}_\odot$$

Hydrogen fusion

hydrogen fusion

$$4\,{}^1\mathrm{H} \rightarrow {}^4\,\mathrm{He} + 2e^+ + 2\nu_e \quad Q = 26.73\,\mathrm{MeV}$$

What reactions are involved? How can one verify that it is so?

Some candidates for proton fusion

$p + n \rightarrow D + \gamma$ Likely unimportant, since there are no free neutrons around

$p + p \rightarrow {}^2\mathrm{He}$ Also does not contribute since ^{2}He is extremely unstable and decays to 2p

Another clue is given by considering the lifetime of an element against destruction (lecture 2):

$$\tau_{pp} = \frac{1}{n_p \langle \sigma v \rangle}$$

The pp-chain

The solution proposed by H. Bethe is a weak interaction

$$p + p \to D + e^+ + \nu$$

Requires an endothermic weak interaction during a p-p scattering event, $p \to n + e^+ \nu$

The cross section is extremely small and impossible to measure in the lab, nevertheless it can be inferred from theory

related to overlap integral between the initial pp and final D wave functions

phase-space factor

Heavy-meson related correction to nuc. force

$$S(0) = 6\pi^2 m_{\rm p} c\alpha \ln 2 \frac{\Lambda^2}{\gamma_D^3} \frac{G_A}{G_V} \frac{f_{pp}^2}{(ft)_{0^+ \to 0^+}} (1+\delta)^2 \simeq 4 \times 10^{-25}\,\mathrm{MeV\ barns}$$

Fine structure constant

Axial and Fermi vector weak coupling constants

phase factor for super allowed 0^+0^+ transitions

Net effect: 0.420 MeV of kinetic energy + annihilation of e^+

$$Q = 1.442\,\mathrm{MeV}, \quad \bar{Q}_\nu = 0.265$$

Lifetime for the Sun:

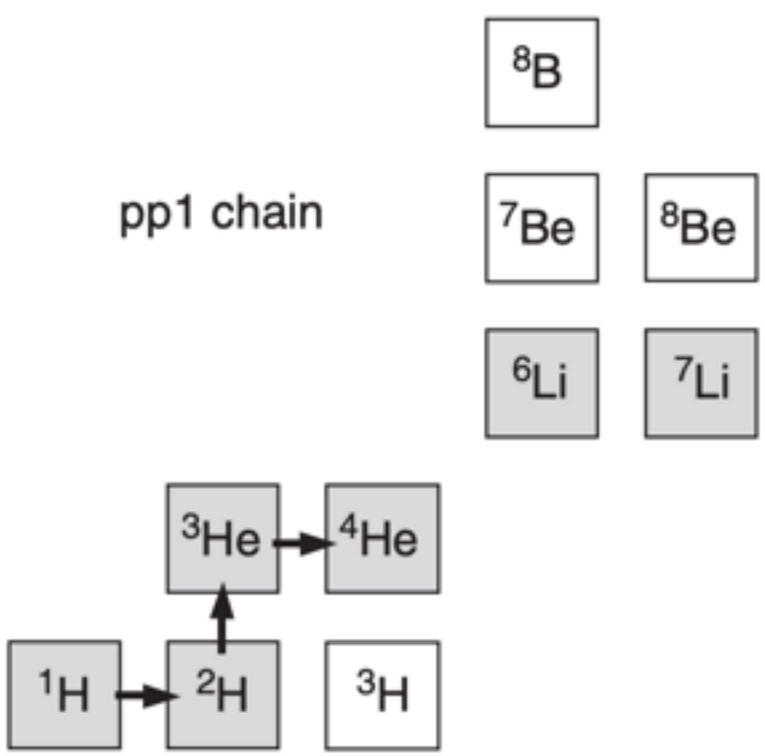
$$\tau_{pp} = \frac{1}{n_p \langle \sigma v \rangle} \simeq \frac{1}{6^{25} \times 8^{-44}} \simeq 10^{10}\,\mathrm{yr}$$

pp-I

The D produced can interact with a lot of things extremely fast. All possibilities need to be considered, but in reality, only a few reactions dominate

reaction:	rate:	Q (MeV)	$\langle E_\nu \rangle$ (MeV)
$H + H \to D + e^+ + \nu$	$r_{pp} = \frac{1}{2} H^2 \langle \sigma v \rangle_{pp}$	1.442	0.265
$D + H \to {}^3He + \gamma$	$r_{pD} = H\,D \langle \sigma v \rangle_{pD}$	5.493	
${}^3He + {}^3He \to {}^4He + 2\,H$	$r_{33} = \frac{1}{2} ({}^3He)^2 \langle \sigma v \rangle_{33}$	12.860	

The evolution of element abundances, considering this chain only, are governed by the following equations



$$\frac{dH}{dt} = -H^2 \langle \sigma v \rangle_{pp} - HD \langle \sigma v \rangle_{pD} + ({}^3He)^2 \langle \sigma v \rangle_{33}$$

$$\frac{dD}{dt} = -0.5H^2 \langle \sigma v \rangle_{pp} - HD \langle \sigma v \rangle_{pD}$$

$$\frac{d^3He}{dt} = HD \langle \sigma v \rangle_{pD} - ({}^3He)^2 \langle \sigma v \rangle_{33}$$

$$\frac{d^4He}{dt} = 0.5({}^3He)^2 \langle \sigma v \rangle_{33}$$

pp-I: evolution of abundances and equilibria

D is both created and destroyed, with its abundance being determined by two competing terms:

$$\frac{dD}{dt} = -0.5\mathrm{H}^2\langle\sigma v\rangle_{pp} - \mathrm{HD}\langle\sigma v\rangle_{pD}$$

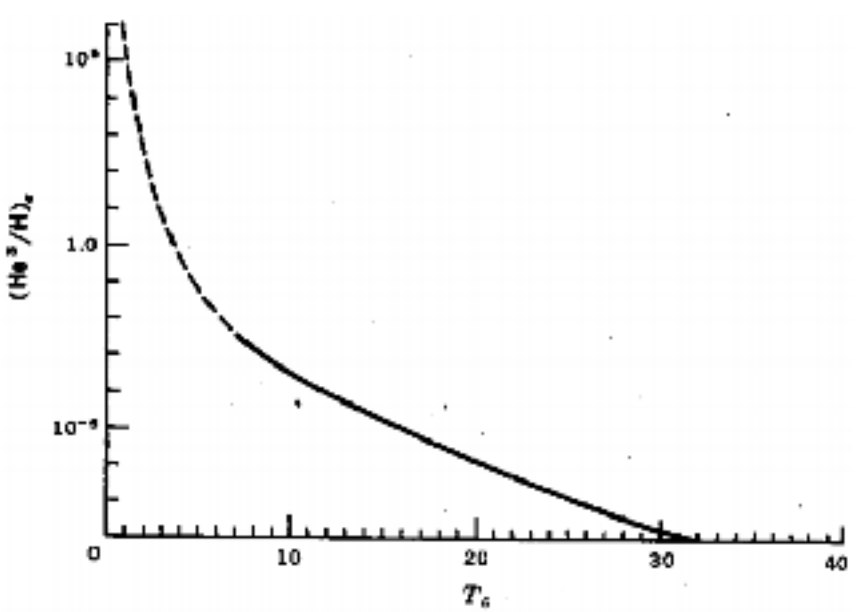
In equilibrium:

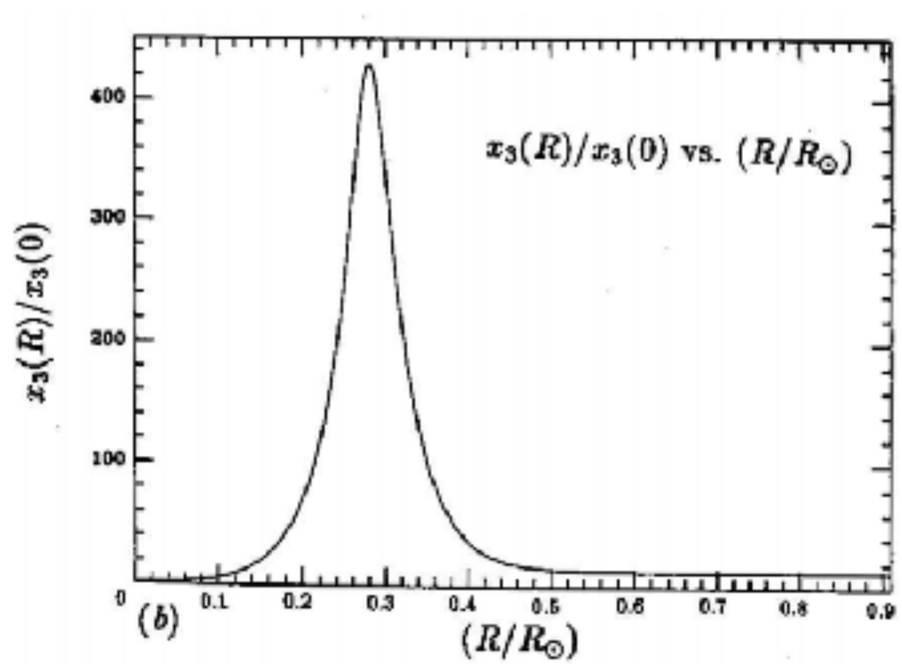
$$\left(\frac{D}{H}\right)_e = \frac{\langle\sigma v\rangle_{pp}}{2\langle\sigma v\rangle_{pD}} \simeq 10^{-18}$$

$\tau_D \sim 1.6\,s$ in the core, which means that equilibrium is reached within seconds

If D in equilibrium then for ^{3}He:

$$\frac{d^3\mathrm{He}}{dt} = \mathrm{HD}\langle\sigma v\rangle_{pD} - (^3\mathrm{He})^2\langle\sigma v\rangle_{33} = \frac{\mathrm{H}^2}{2}\langle\sigma v\rangle_{pp} - (^3\mathrm{He})^2\langle\sigma v\rangle_{33} \Rightarrow \left(\frac{^3\mathrm{He}}{H}\right)_e = \left(\frac{\langle\sigma v\rangle_{pp}}{2\langle\sigma v\rangle_{33}}\right)^{1/2}$$





pp-I: evolution of abundances and equilibria

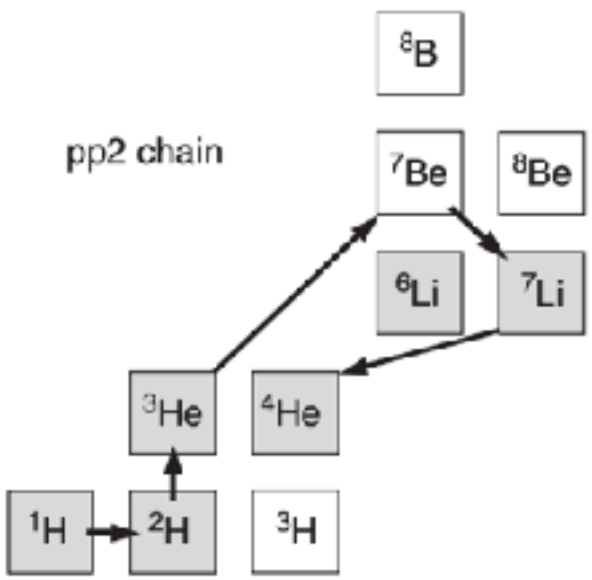
If both D and ^{3}He in equilibrium, then $d\mathrm{H}/dt = -2r_{pp}$ and $d^4\mathrm{He}/dt = \frac{1}{2}r_{pp}$

Conclusion: the rate of the entire chain is set by the pp reaction

$$\epsilon_{ppI} = Q_{\mathrm{eff}}\frac{r_{pp}}{2\rho} = 26.21\,\mathrm{MeV}\frac{r_{pp}}{2\rho}$$

ppII and ppIII

require He, which is only used as a catalyst



$$^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H} + e^{+} + \nu$$
$$^{2}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + \gamma$$

$$^{3}\mathrm{He} + {}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + 2\,{}^{1}\mathrm{H}$$

ppI

competing at high T

$$^{3}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{7}\mathrm{Be} + \gamma$$

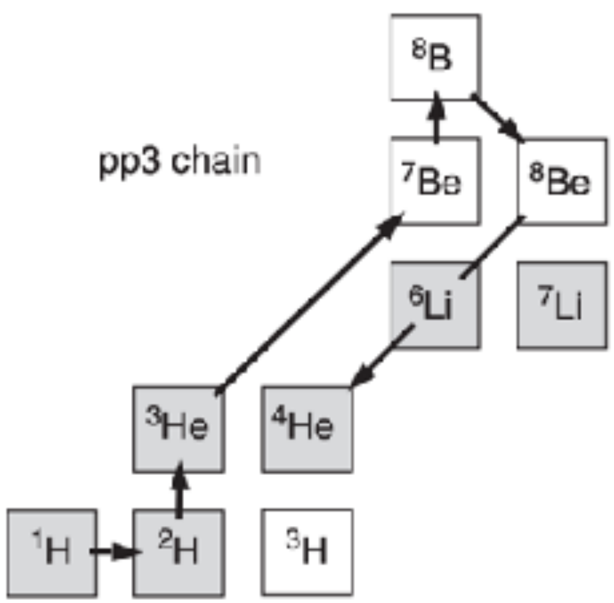
τ~0.3yr

$$^{7}\mathrm{Be} + e^{-} \rightarrow {}^{7}\mathrm{Li} + \nu$$
$$^{7}\mathrm{Li} + {}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He}$$

ppII

$$^{7}\mathrm{Be} + {}^{1}\mathrm{H} \rightarrow {}^{8}\mathrm{B} + \gamma$$
$$^{8}\mathrm{B} \rightarrow {}^{8}\mathrm{Be}^{*} + e^{+} + \nu$$ τ~1s
$$^{8}\mathrm{Be}^{*} \rightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He}$$ τ~10^{-22}s

ppIII



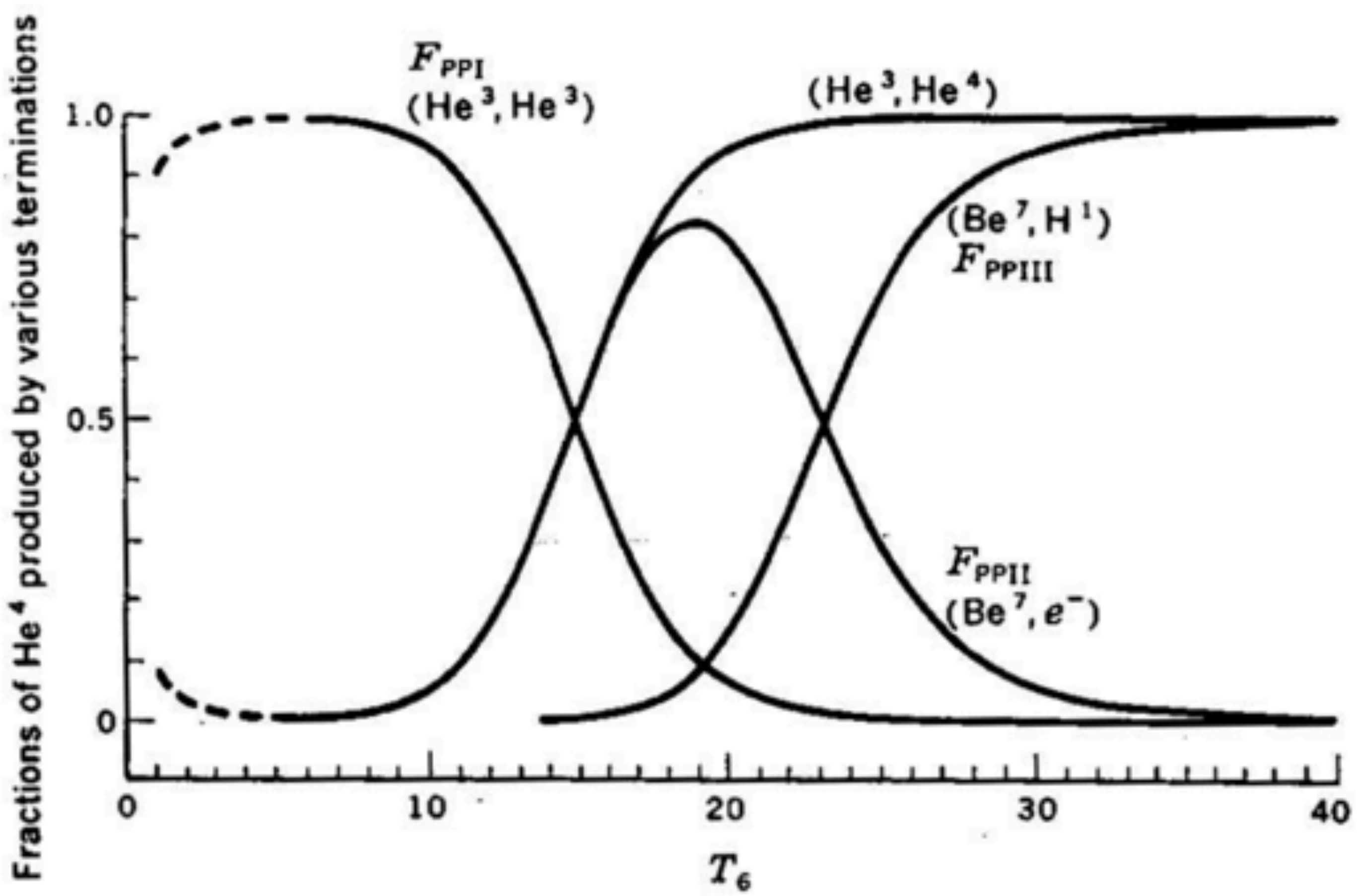
reaction	Q (MeV)	$\langle E_\nu \rangle$ (MeV)	$S(0)$ (keV barn)	dS/dE (barn)	τ (yr)
$^{1}\mathrm{H}(p, e^{+}\nu)^{2}\mathrm{H}$	1.442	0.265	3.94×10^{-22}	4.61×10^{-24}	10^{10}
$^{2}\mathrm{H}(p, \gamma)^{3}\mathrm{He}$	5.493		2.5×10^{-4}	7.9×10^{-6}	10^{-8}
$^{3}\mathrm{He}(^{3}\mathrm{He}, 2p)^{4}\mathrm{He}$	12.860		5.18×10^{3}	-1.1×10^{1}	10^{5}
$^{3}\mathrm{He}(\alpha, \gamma)^{7}\mathrm{Be}$	1.587		5.4×10^{-1}	-3.1×10^{-4}	10^{6}
$^{7}\mathrm{Be}(e^{-}, \nu)^{7}\mathrm{Li}$	0.862	0.814			10^{-1}
$^{7}\mathrm{Li}(p, \alpha)^{4}\mathrm{He}$	17.347		5.2×10^{1}	0	10^{-5}
$^{7}\mathrm{Be}(p, \gamma)^{8}\mathrm{B}$	0.137		2.4×10^{-2}	-3×10^{-5}	10^{2}
$^{8}\mathrm{B}(e^{+}\nu)^{8}\mathrm{Be}^{*}(\alpha)^{4}\mathrm{He}$	18.071	6.710			10^{-8}

ppII and ppIII

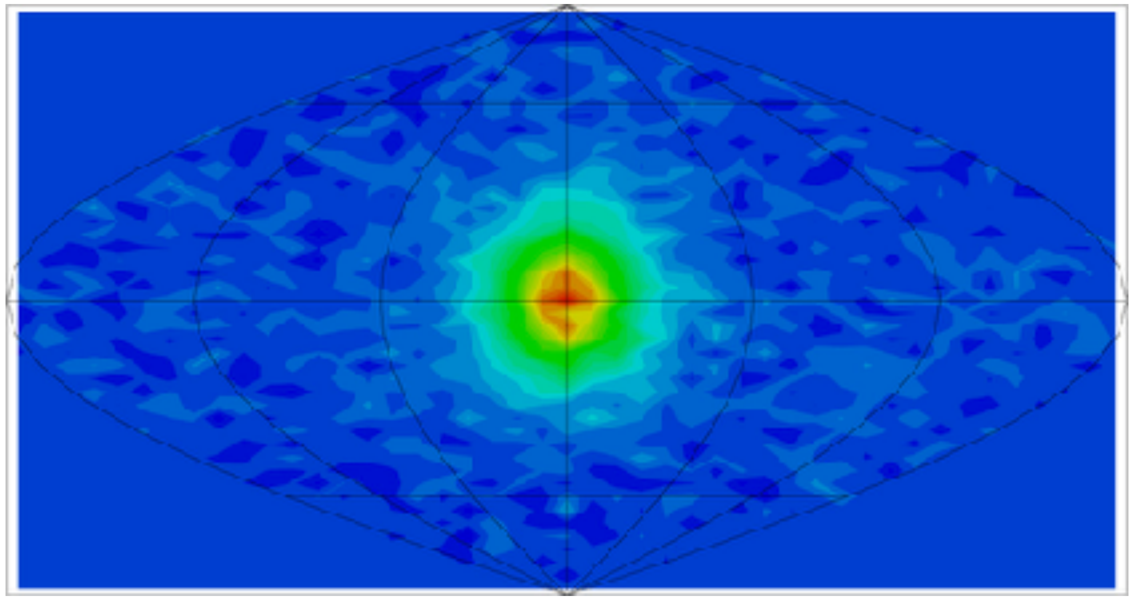
Branching ratio is determined by the two weakest reactions in the respective chains

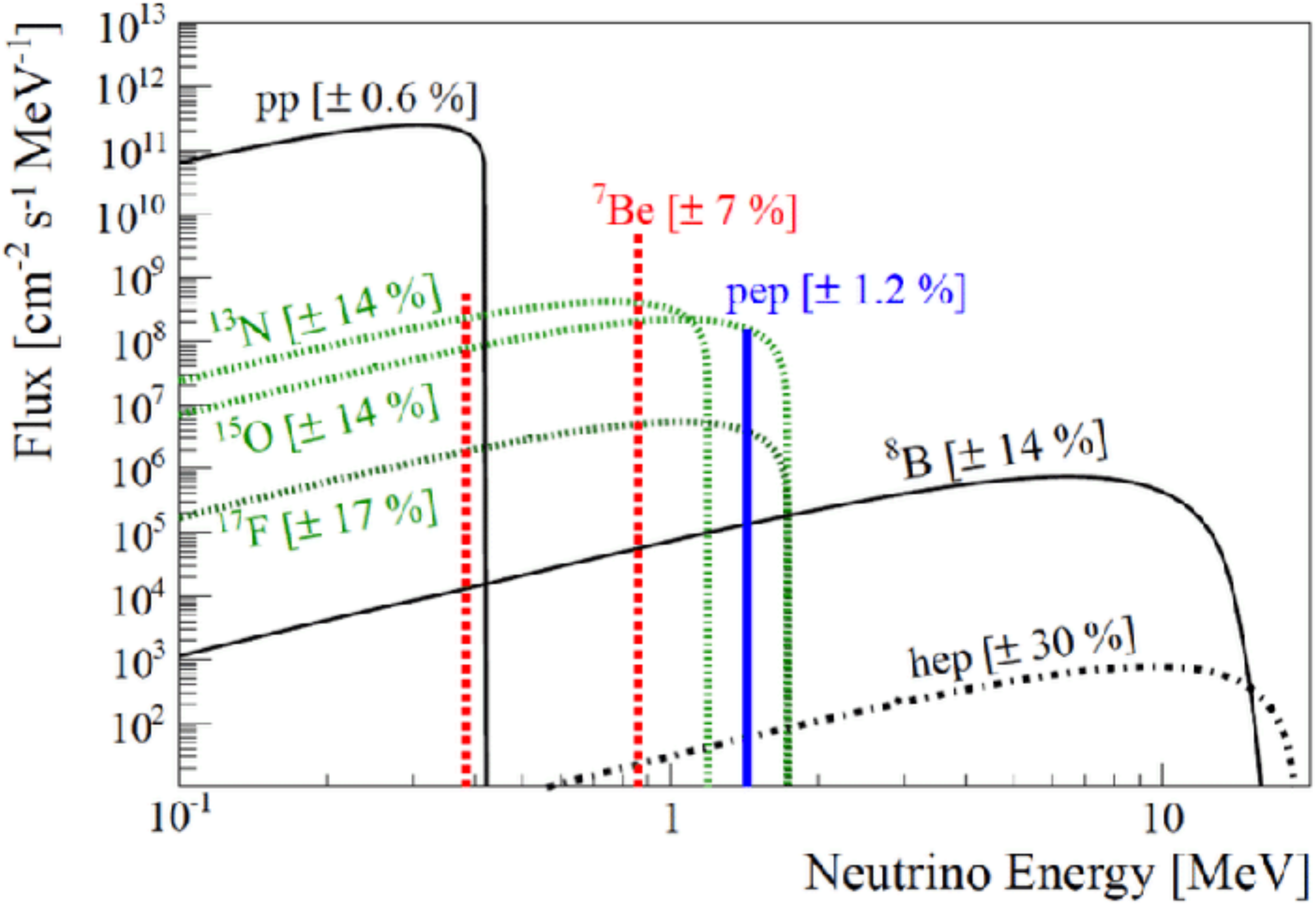
$$\frac{f_{ppI}}{f_{ppII} + f_{ppIII}} = \frac{r_{33}}{r_{34}} = \frac{\langle\sigma v\rangle_{33}}{2\langle\sigma v\rangle_{34}} \frac{{}^3\mathrm{He}}{{}^4\mathrm{He}}$$

ppI dominates at low He abundance and low temperature. ppII dominates when hydrogen gets depleted. ppIII dominates at high temperatures.
The energy released in each chain is the same, however the neutrino losses are different



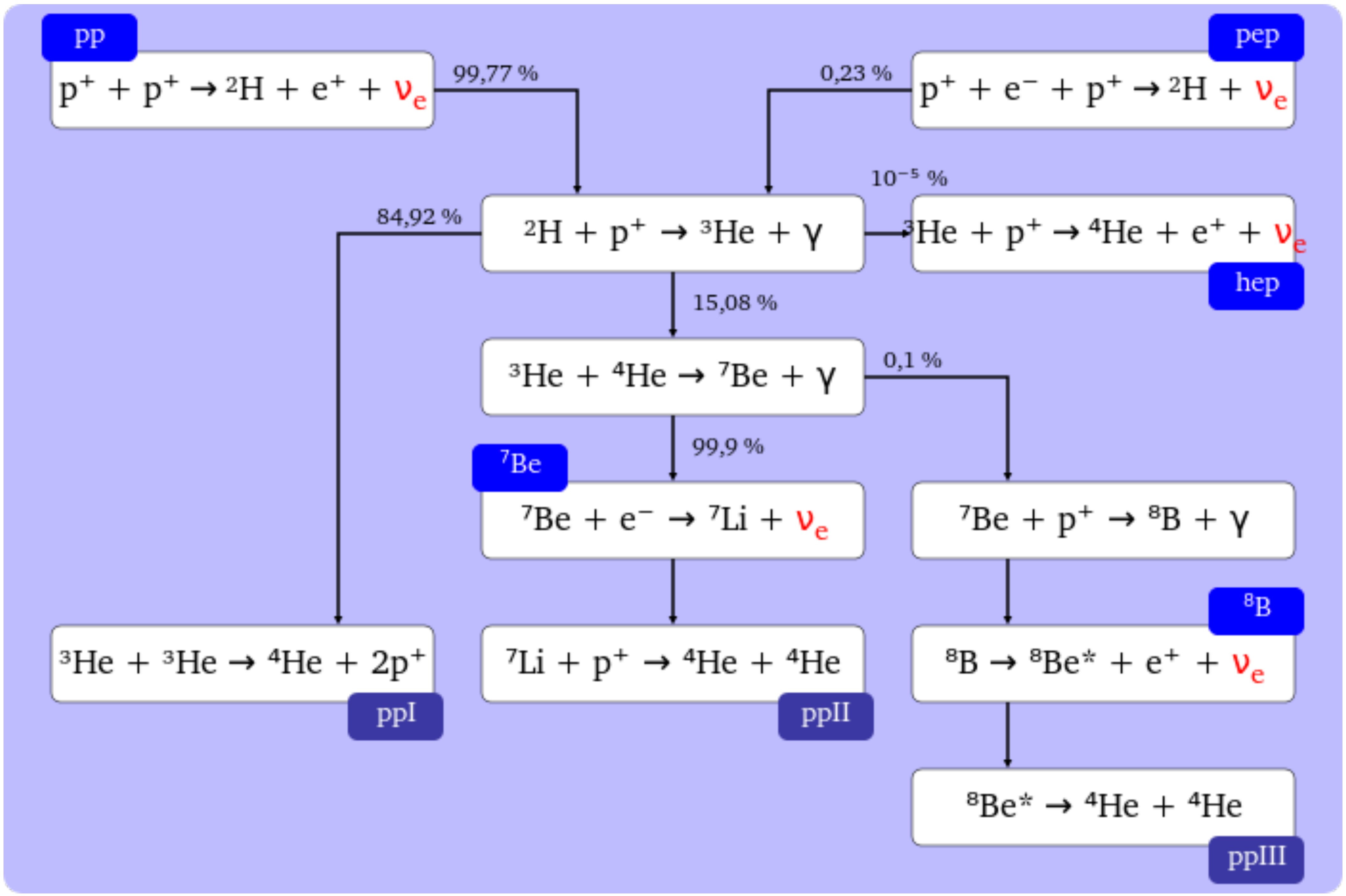
solar neutrinos





Super-Kamiokande

solar neutrinos



coming up

CNO, NeNa, MgAl cycles, He burning, advanced burning