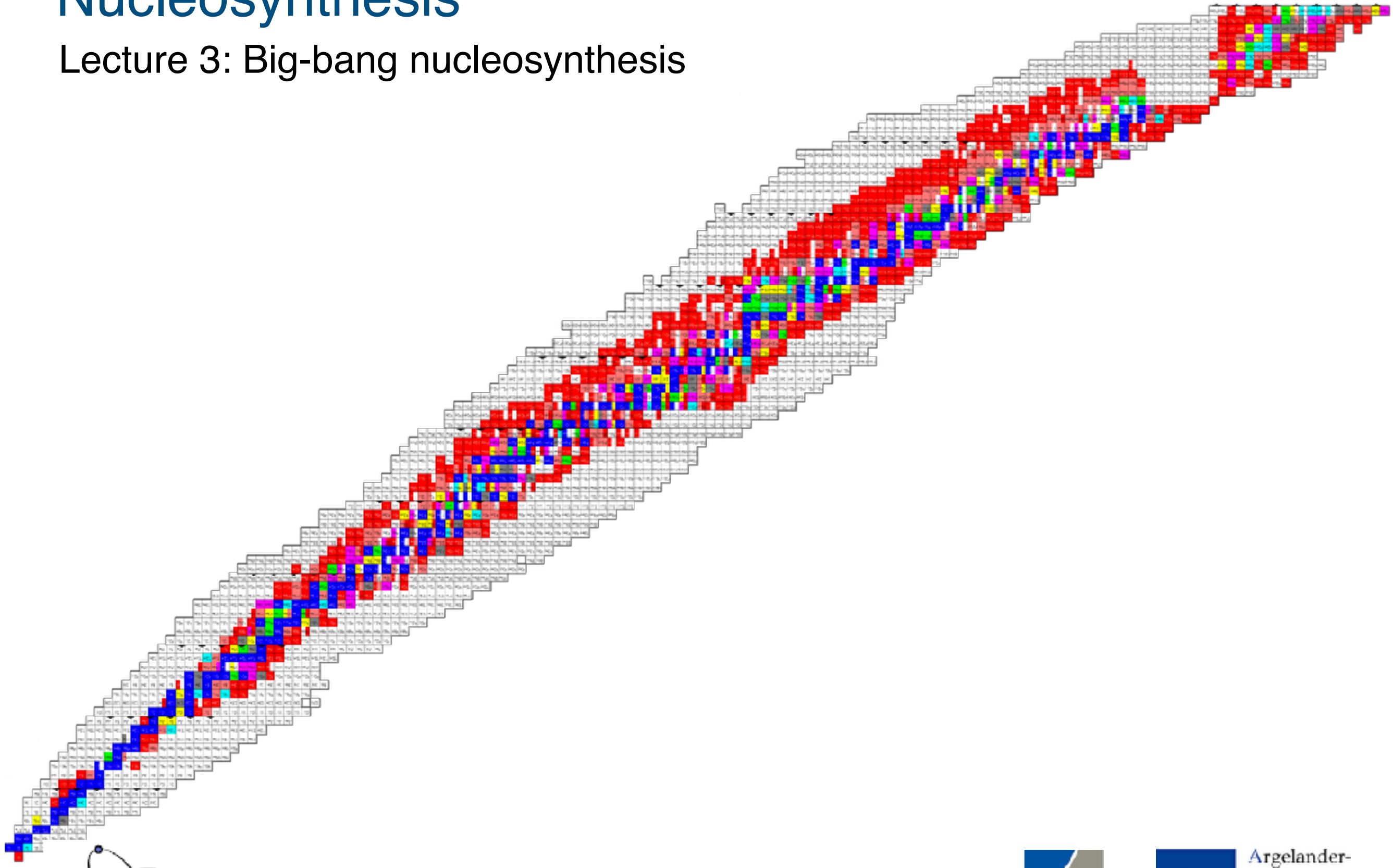


Nucleosynthesis

Lecture 3: Big-bang nucleosynthesis



Bonn-Cologne Graduate School
of Physics and Astronomy

Overview

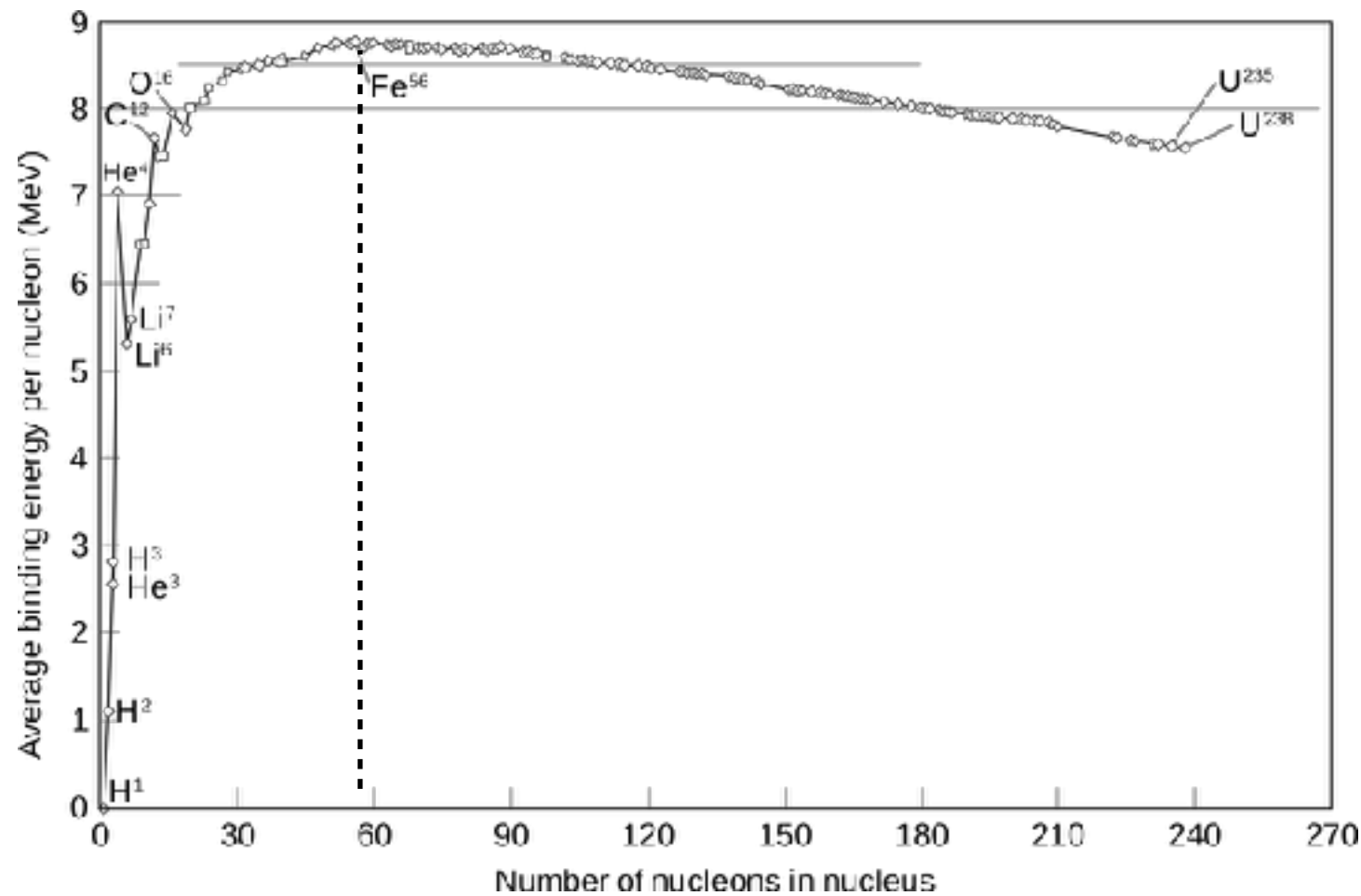
• Lecture 1: Introduction & overview	April 18
• Lecture 2: Thermonuclear reactions	April 25
• Lecture 3: Big-bang nucleosynthesis	May 2
• Lecture 4: Thermonuclear reactions inside stars — I (H-burning)	May 7
• Lecture 5: Thermonuclear reactions inside stars — II (advanced burning)	May 16
• Lecture 6: Neutron-capture and supernovae — I	May 23
• Lecture 7: Neutron-capture and supernovae — II	June 6
• Lecture 8: Thermonuclear supernovae	June 13
• Lecture 9: Li, Be and B	July 4
• Lecture 10: Galactic chemical evolution and relation to astrobiology	July 11
 Paper presentations I	 June 21
Paper presentations II	June 27

Overview of previous lecture

Energetics of nuclear reactions

$$Q \equiv E_{Y'b'} - E_{\alpha'X'} = (\Delta M_{\alpha'} + \Delta M_{X'} - \Delta M_{Y'} - \Delta M_{b'})c^2$$

Exothermic reaction $Q > 0$
Endothermic reaction $Q < 0$



Nuclear reaction rate

$$r_{\alpha X} = N_{\alpha} N_X \int_0^{\infty} v \sigma(v) \phi(v) dv = N_{\alpha} N_X \langle \sigma v \rangle \quad \text{with} \quad \int \phi(v) dv = 1$$

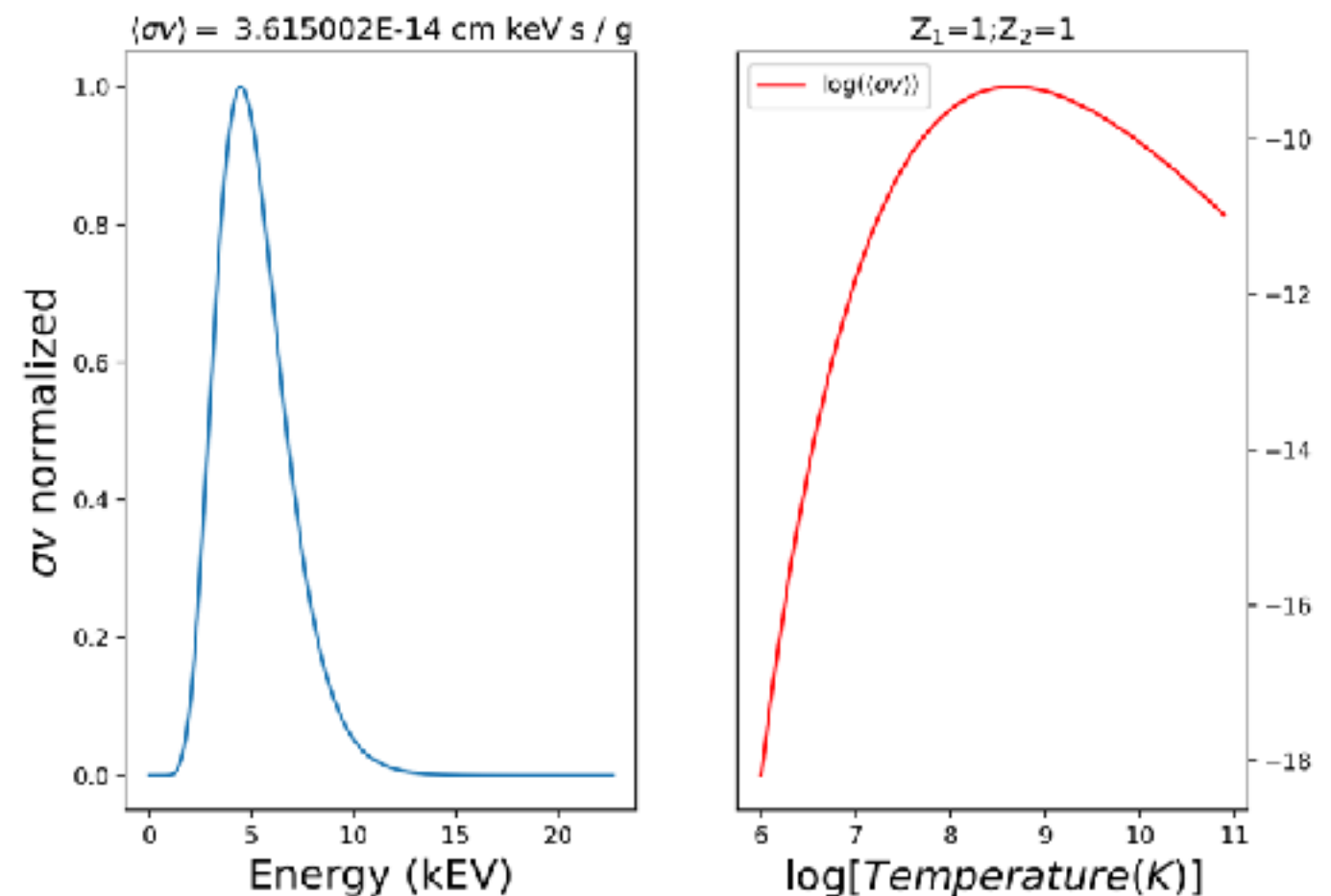
Overview of previous lecture

Ways to evaluate $\langle\sigma v\rangle$

- lab measurements (usually high energies, strong and E/M reactions)
- nuclear theory (approximate) + simplifications (e.g., away from resonances: constant astrophysical factor, dependence only on Gamow penetration and thermal energy distribution)

Three salient properties of $\langle\sigma v\rangle$

1. Depends on nature of interaction
2. Significant only for narrow range of energies (Gamow peak for constant T)
3. Strongly varying function of temperature



At sufficiently high temperatures, forward and reverse reactions balance. When this happens, abundances are determined by statistical physics, $N(A, Z) \propto \exp[Q(A, Z)/kT]$

When this happens for all possible reactions, **nuclear statistical equilibrium** is achieved

Overview of previous lecture

Once rates and conditions (in T.E.: composition, temperature and density) are known, we can start cooking up the elements

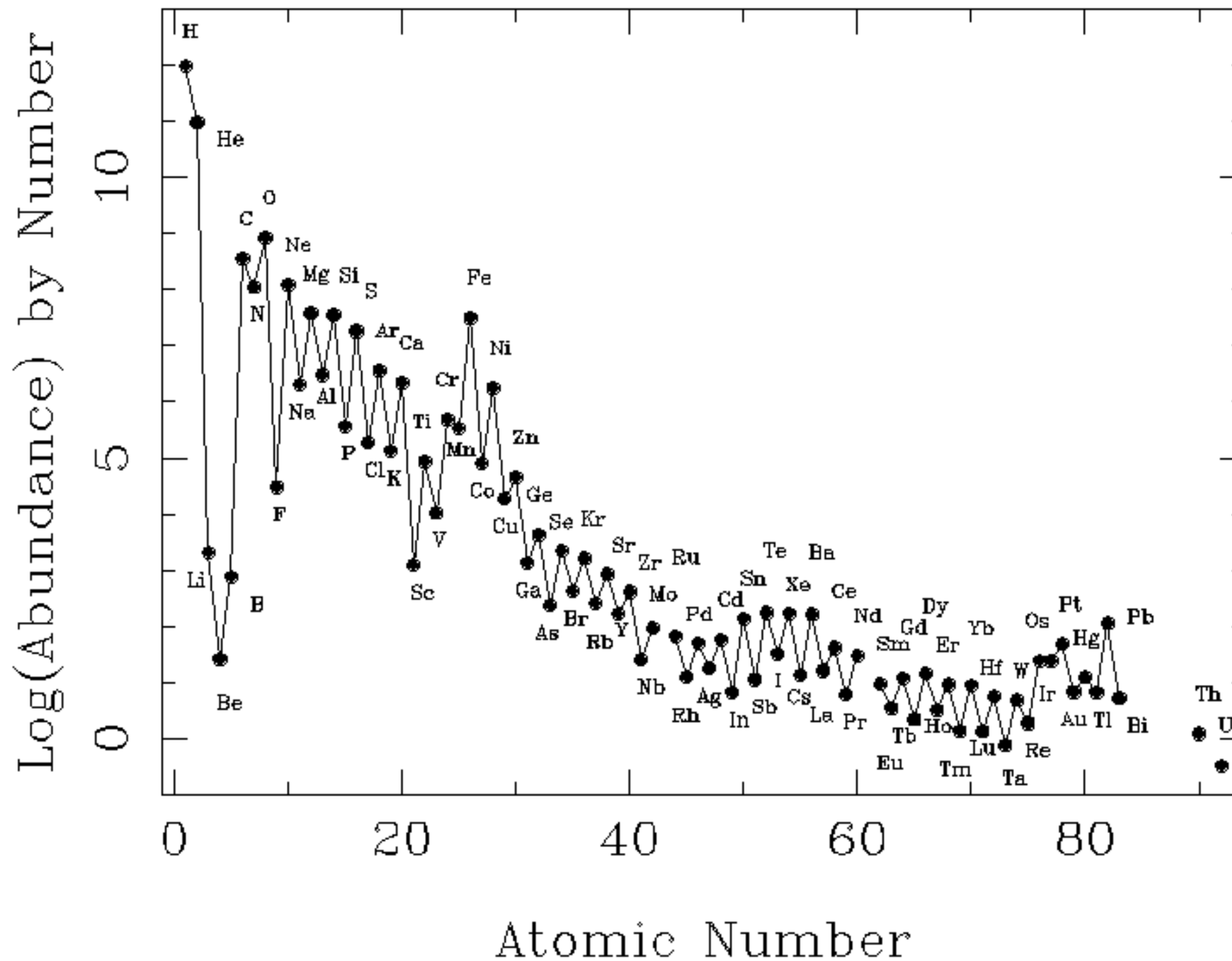
$$\frac{dX_i}{dt} = A_i \frac{m_u}{\rho} \left(- \sum r_{\text{reactions that destroy } i} + \sum r_{\text{reactions that create } i} \right)$$

Energy generation rate:

$$\epsilon_{\alpha X} = \frac{Q_{\alpha X}}{m_u^2} \frac{X_\alpha X_X}{A_\alpha A_X} \rho \langle \sigma v \rangle_{\alpha X} \simeq \epsilon_0 X_\alpha X_X \rho T^\nu$$

The origin of light elements (isotopes of H, He and Li)

Logarithmic SAD Abundances: $\text{Log}(\text{H}) = 12.0$



The origin of light elements (isotopes of H, He and Li)

**H and He isotopes account for ~98% of the baryonic mass in the Universe.
Can they be made in stars?**

No.

- Only ~10% of a star is converted to He via fusion
- Most of the He is subsequently destroyed in reactions or retained in stellar remnants
- Observationally, He abundance does not vary significantly with metallicity
- Deuterium is destroyed during the pre-ms via $D(p, \gamma)^3\text{He}$ which has a large cross section

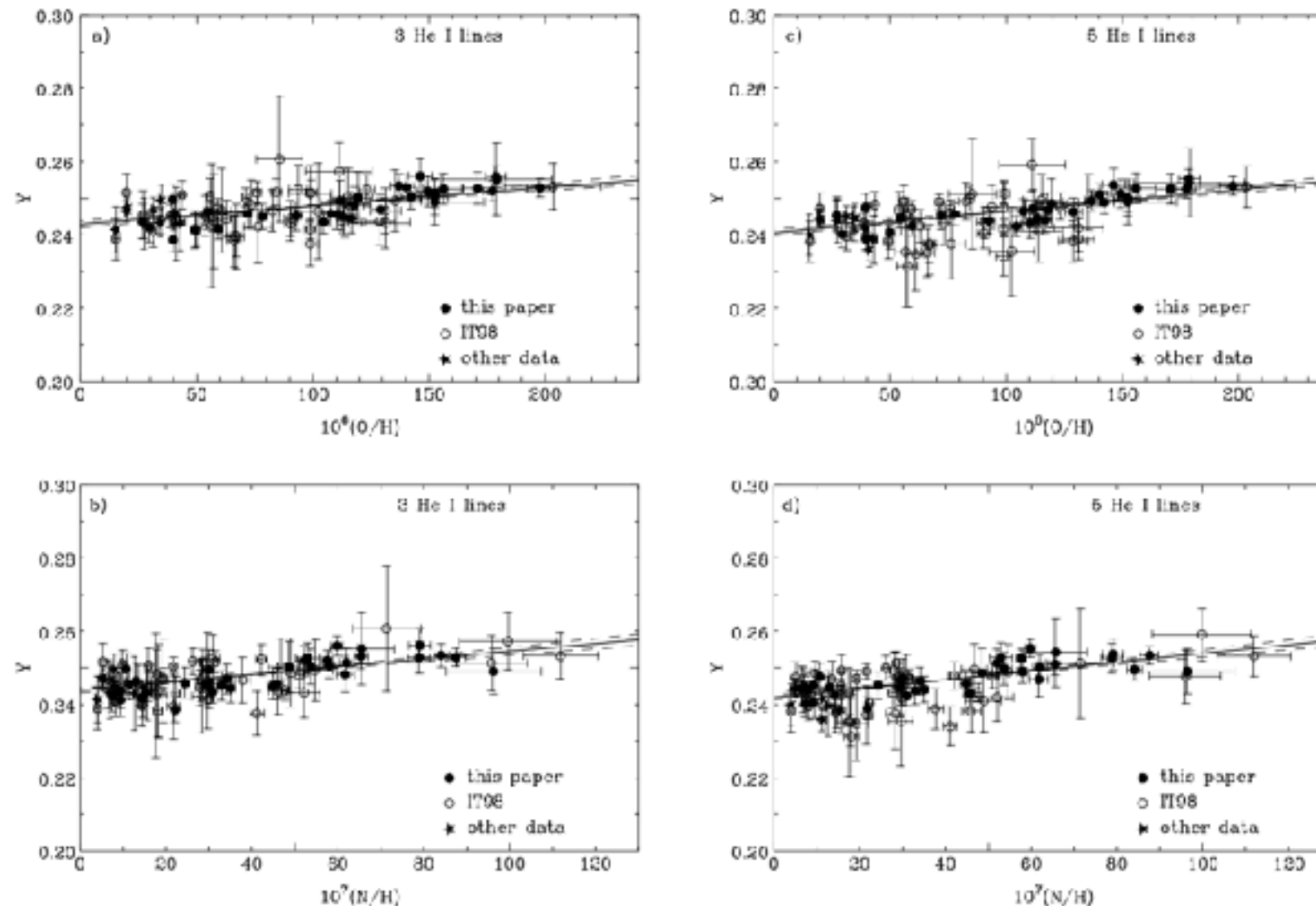


FIG. 2.—Linear regressions of the helium mass fraction Y vs. oxygen and nitrogen abundances for a total of 82 H II regions in 76 BCGs. In (a) and (b), Y was derived using the three $\lambda\lambda 4471$, 5876 , and 6678 He I lines, and in (c) and (d), Y was derived using the five $\lambda\lambda 3889$, 4471 , 5876 , 6678 , and 7065 He I lines.

The origin of light elements (isotopes of H, He and Li)

Hypothesis: Light elements (particularly H and He isotopes) were created shortly after the Big Bang under very special conditions

The Alpher, Bethe & Gamow (1948) paper (aka $\alpha\beta\gamma$ paper)

The Origin of Chemical Elements

R. A. ALPHER*

*Applied Physics Laboratory, The Johns Hopkins University,
Silver Spring, Maryland*

AND

H. BETHE

Cornell University, Ithaca, New York

AND

G. GAMOW

The George Washington University, Washington, D. C.

February 18, 1948

Spoiler: this paper is wrong

Big Bang Nucleosynthesis

Nuclear fusion is only possible if the Universe was hot ($\sim \text{MeV}$) enough for a sufficiently long period of time. The Universe is expanding (and cooling) which means that nucleosynthesis could only occur at very early epochs, when the scale factor was of order 10^{-10} . Only at such early times, the rates of some reactions could be much faster than the expansion rate (thermal equilibrium)

Ingredients needed to construct a quantitative theory

- We need to know:
1. Temperature (and density) as a function of time
 2. Primordial composition of elementary particles
 3. Reaction rates between all components

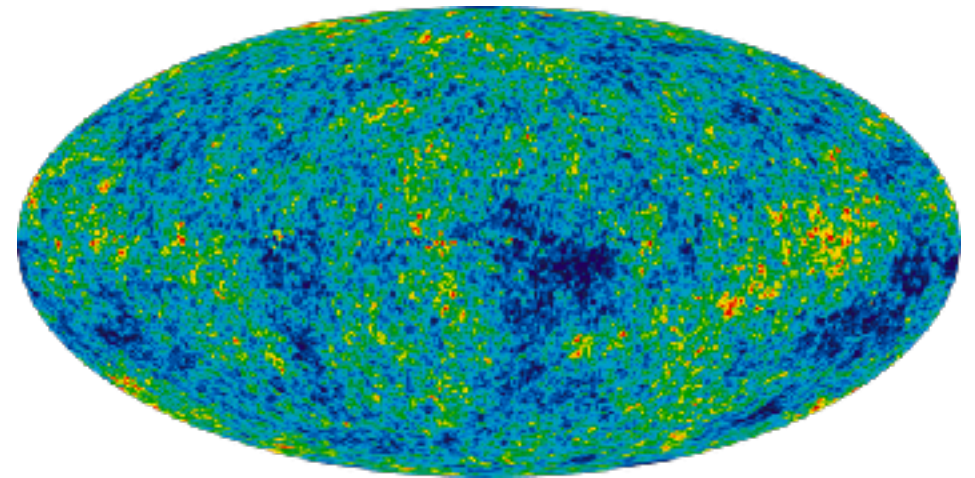
Cosmology

Assumptions

- Cosmological principle (the Universe is isotropic and homogeneous)
- General relativity describes gravity at all scales
- Standard model of particle physics

Observables

$$v_{\text{expansion}} = D \times H_0; H_0 = (67.4 \pm 0.5) \text{ km s}^{-1} \quad +$$



Adiabatic expansion

$$z \equiv \frac{\lambda - \lambda_0}{\lambda_0}, T = T_{\text{CMB}} \left(\frac{a_0}{a} \right) = 2.73 \text{ K} \times (1 + z) \quad \text{so the Universe was indeed hot very early on}$$

General relativity

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3} G \rho + \frac{\Lambda}{3} - \frac{kc^2}{a^2} = \frac{8\pi}{3} G \left(\frac{\rho_{\text{baryon}}}{a^3} + \frac{\rho_{\text{rad}}}{a^4} \right) + \frac{\Lambda}{3} - \frac{kc^2}{a^2}$$

Big Bang Nucleosynthesis

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho + \frac{\Lambda}{3} - \frac{kc^2}{a^2} = \frac{8\pi}{3}G \left(\frac{\rho_{\text{baryon}}}{a^3} + \frac{\rho_{\text{rad}}}{a^4} \right) + \frac{\Lambda}{3} - \frac{kc^2}{a^2}$$

$N \propto a^{-3}$
 $E \propto 1/\lambda \propto 1/a$

so at very early epochs, radiation completely dominates; no need to worry about dark matter/energy

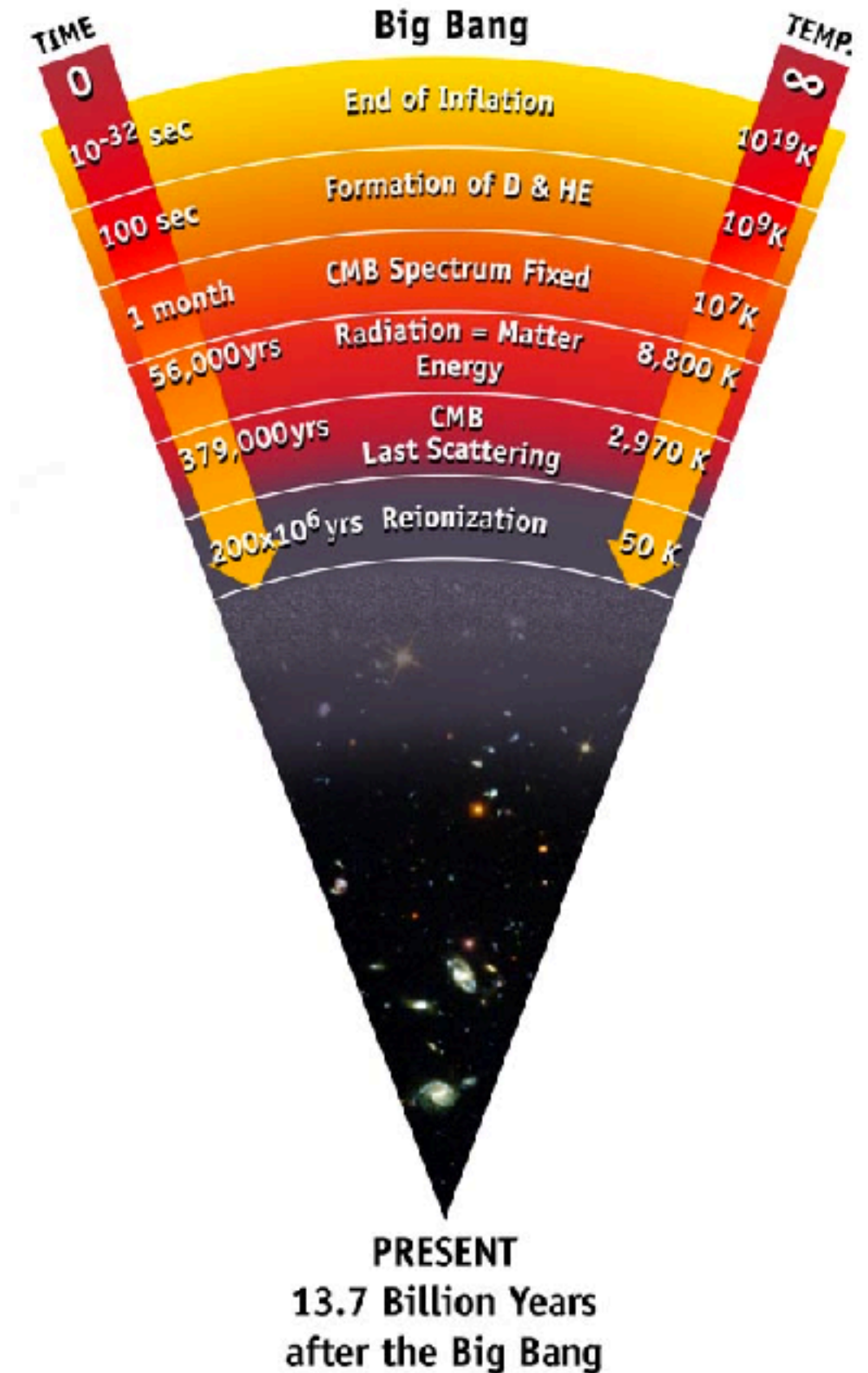
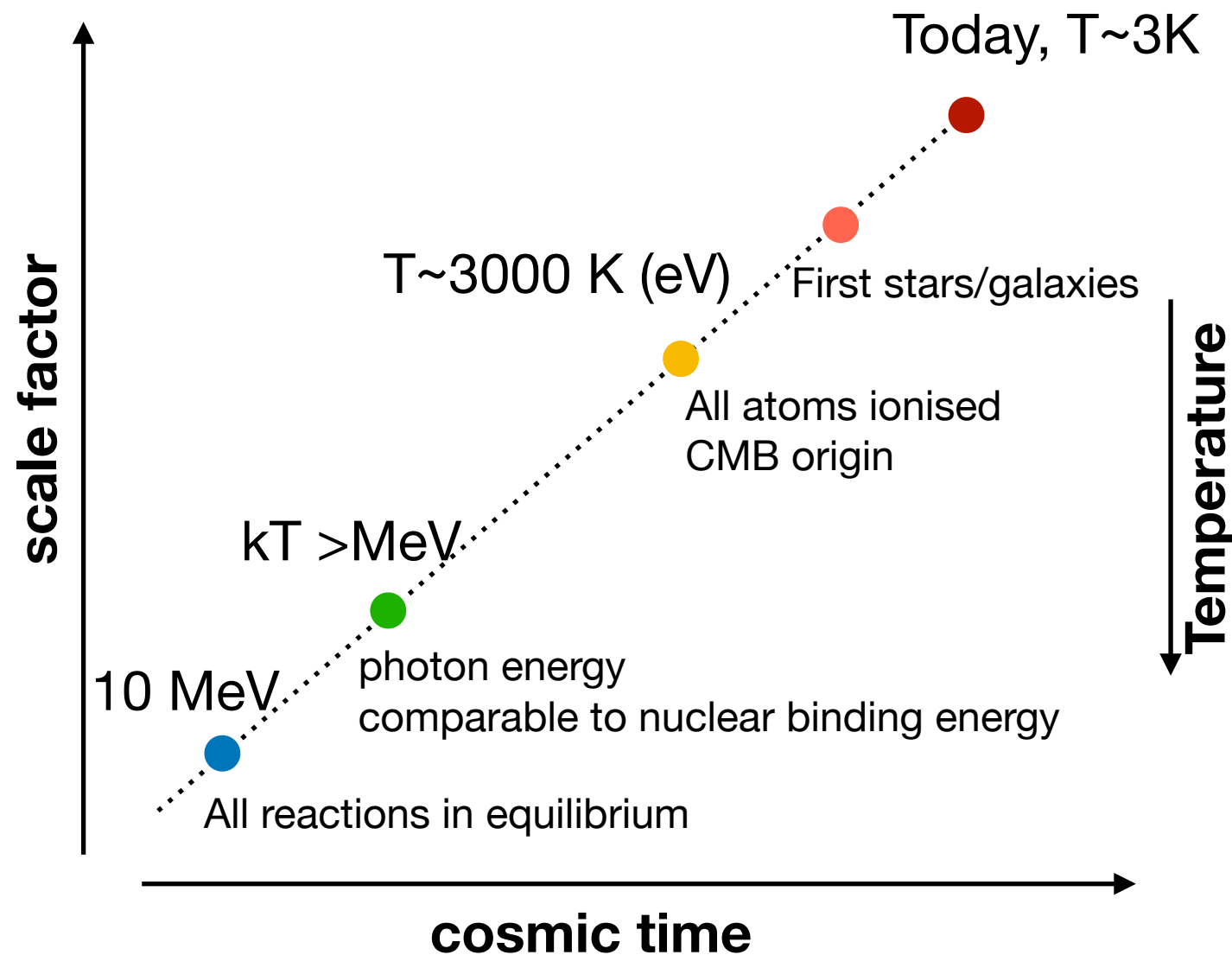
$$a \propto t^{1/2}; \rho_{\text{rad}} \propto T^4 \propto a^{-4} \rightarrow T^2 \propto 1/t$$

This scaling of temperature with cosmic time is very important for nucleosynthesis. It means that the early Universe was a “defect” reactor; reaction rates compete with the expansion rate

A more exact relation will be derived later

Thermal history of the Universe (backwards)

The energy of relativistic particles (photons, neutrinos, electrons, positrons, etc) increased as we move back in cosmic time



baryon to photon ratio, η

The primordial composition influences nucleosynthesis, so it's important to know the number of (anti)baryons and the number of relativistic particles (photons, neutrinos).
How many of each are present?

$T > 10^{13}$ K All particles+anti-particles are in equilibrium. Some examples:

$$e^- e^+ \leftrightarrow 2\gamma; x\bar{x} \leftrightarrow 2\gamma; \nu\bar{\nu} \leftrightarrow 2\gamma; n + \nu \leftrightarrow p + e^-; p + \bar{\nu} \leftrightarrow n + e^+$$

Roughly equal number of baryons and photons: $\eta \equiv \frac{n_b}{n_p} \simeq 1$

At lower temperatures photons are no longer energetic enough to create (anti)baryons. For equal number of particles and anti-particles, this would soon lead to $\eta=0$. Obviously, this is not the case, which means that the number of particles was slightly larger (at least in this part of the Universe). Still $\eta \ll 1$

$T \simeq 1.5 \times 10^{10}$ K neutrinos no longer in equilibrium with e^-e^+

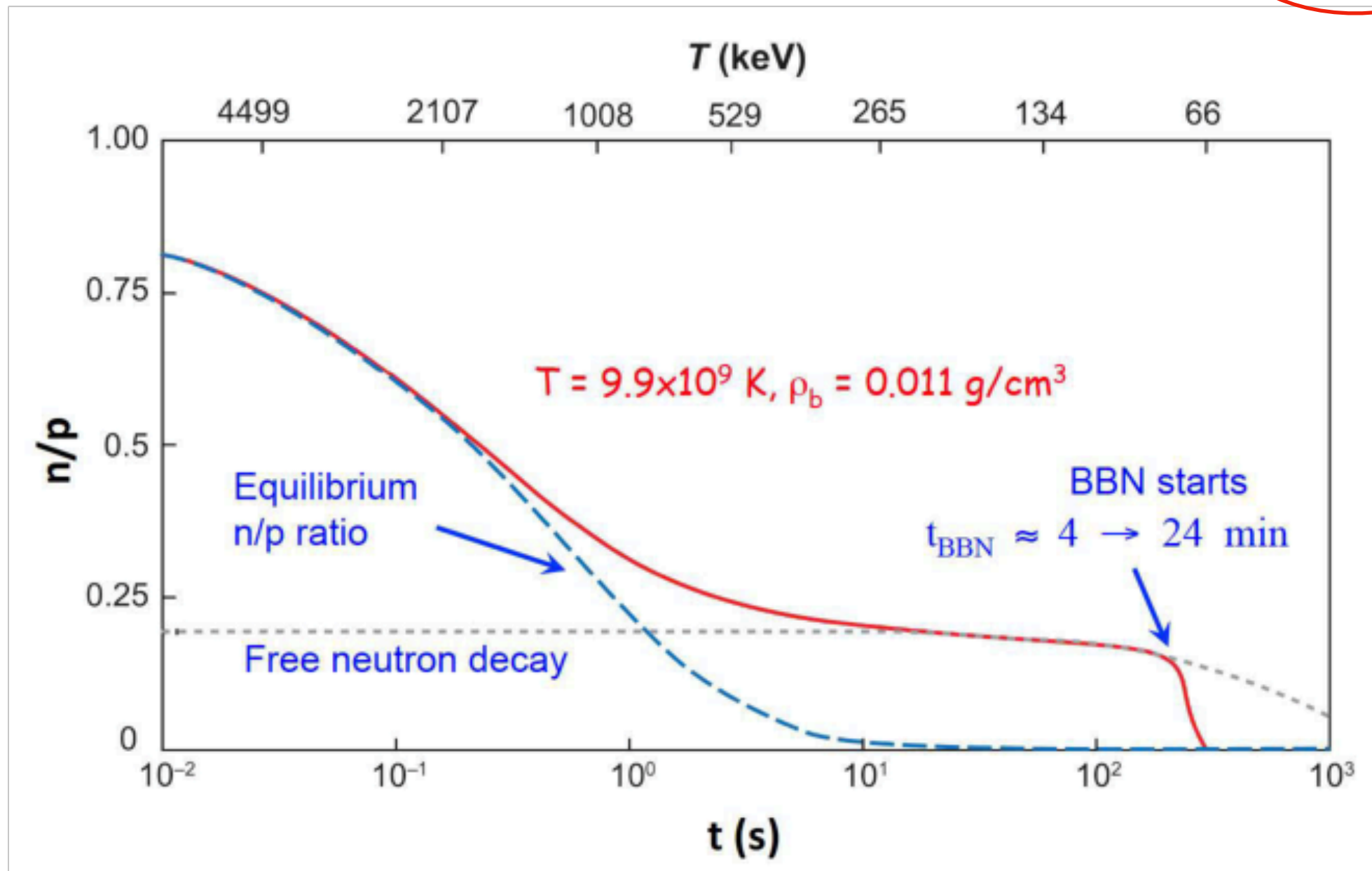
$T \simeq 6 \times 10^9$ K Electron-positron pairs annihilate, energy (more precisely entropy) transferred to photons

Conclusion: shortly after the big bang (and before the onset of nucleosynthesis η became very small. Since this quantity is conserved it remains small (of order 10^{-10}) to the present day. η can be measured directly (exercise 3.3), but for now we shall treat it as a **free parameter**

proton to neutron ratio

Initially, neutrons and protons are in statistical equilibrium

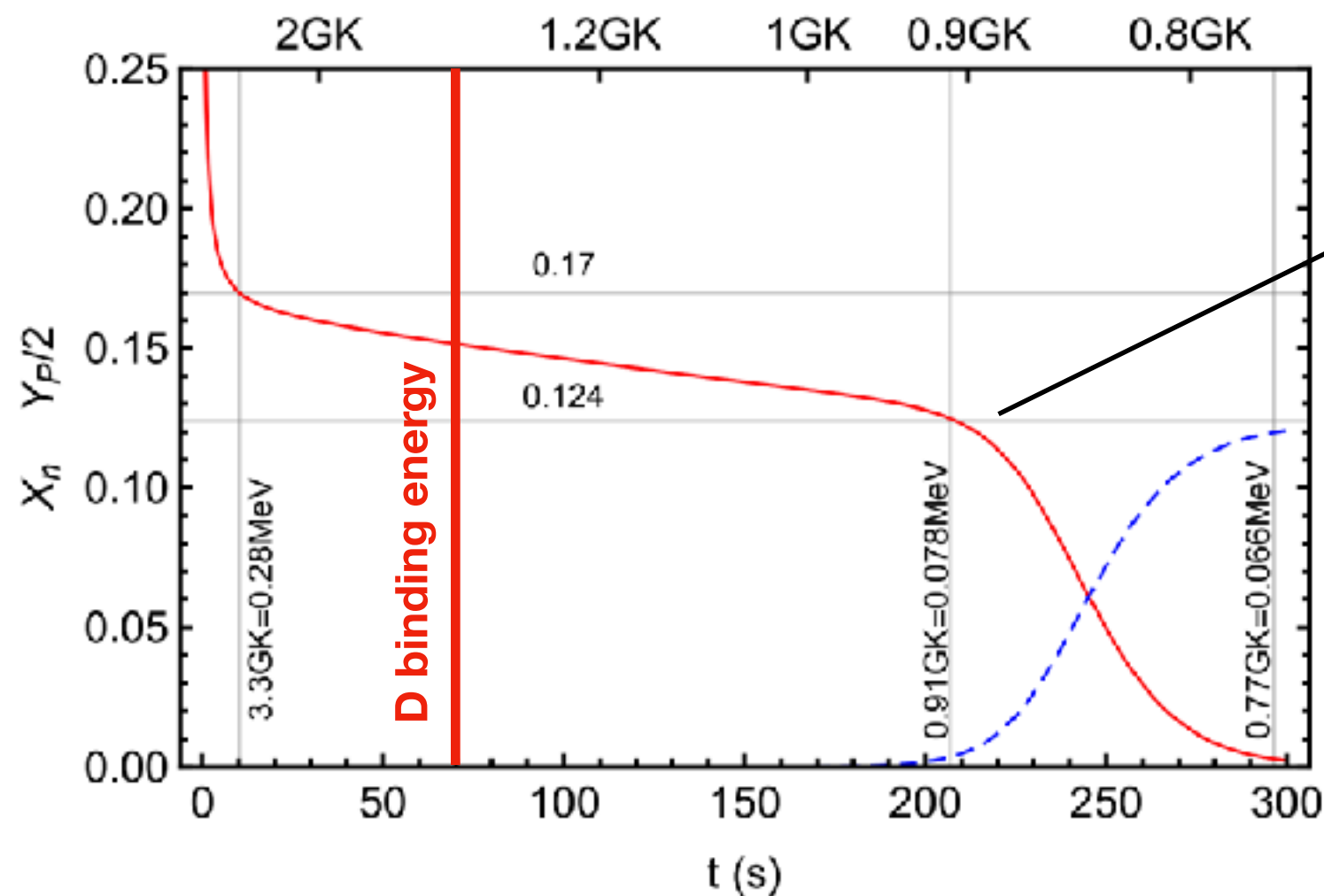
$$n \propto e^{(-938.2 \text{ MeV}/kT)}; p \propto e^{(-939.5 \text{ MeV}/kT)} \rightarrow p \propto e^{(-938.2 \text{ MeV}/kT)} \times e^{(-1.3 \text{ MeV}/kT)}$$



nucleosynthesis begins

Now we know the initial conditions (temperature vs time, η , n/p) for this very special environment. Together with a nuclear reaction network we can thus calculate the evolution of nuclear abundances

The first reaction that happens is $n + p \rightarrow D + \gamma$ which is only possible due to the high abundance of neutrons. Even though the binding energy of D is ~ 2.2 MeV the reaction starts around 1MeV, due to the high number of energetic photons at the tail of the thermal distribution



Eventually almost all neutrons end up in Helium nuclei

Out of 100 nuclei:

88 protons and 12 neutrons

76 protons and 12 Deuterium

76 protons and 6 Helium

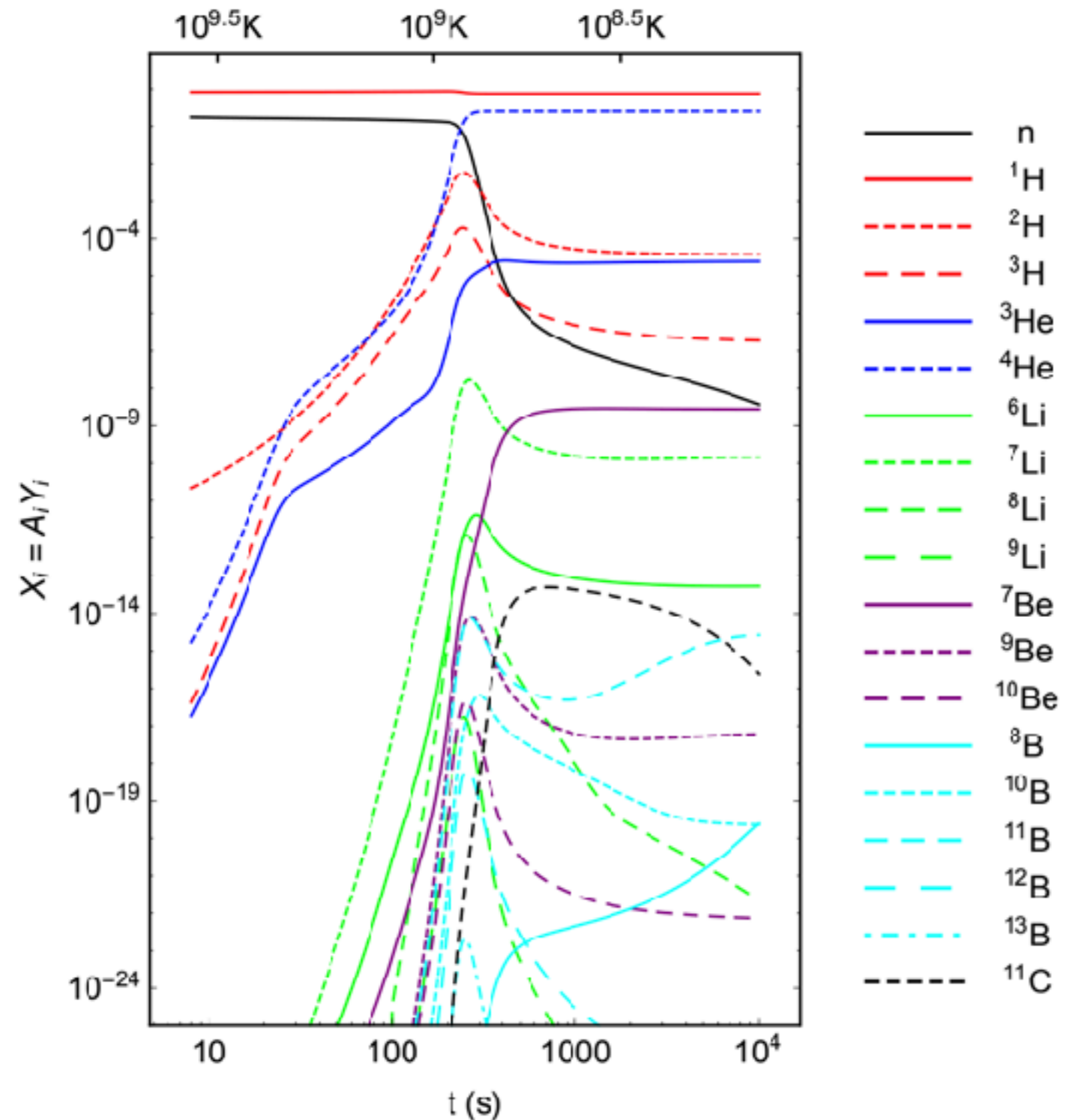
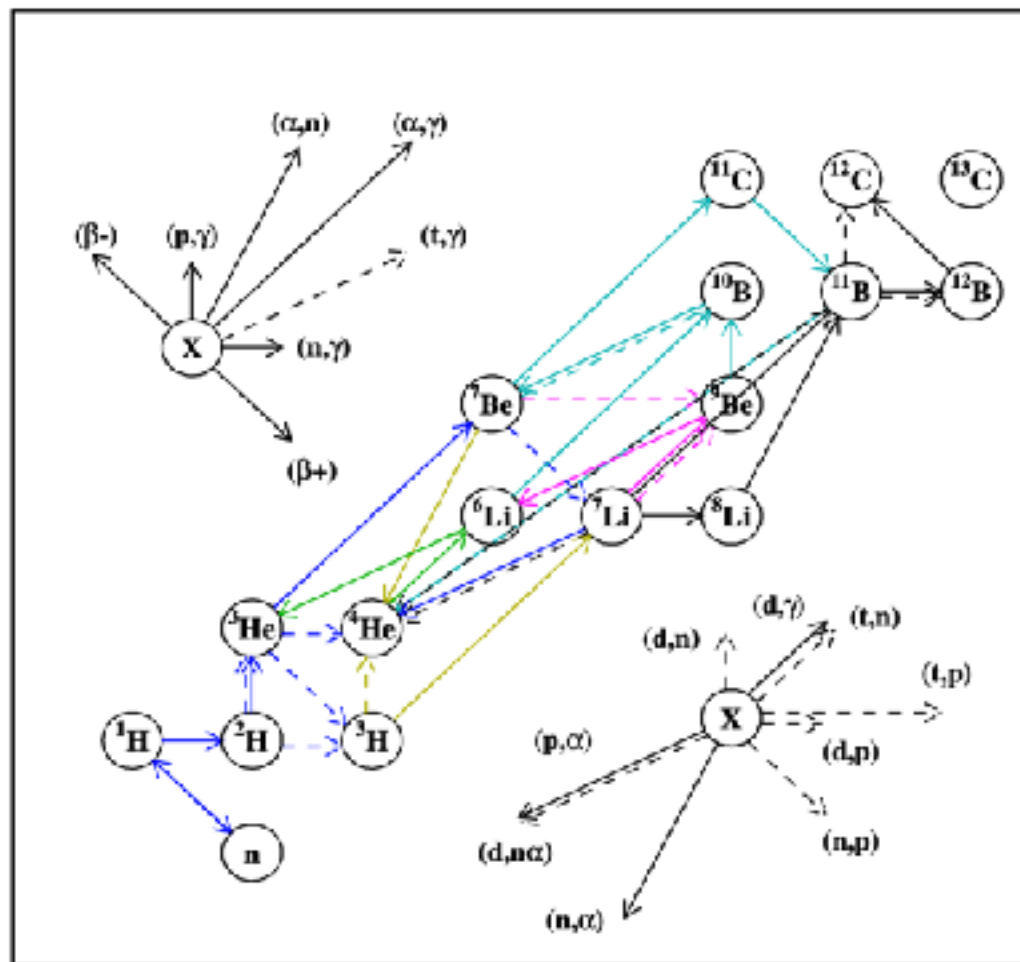
or by mass: 24% Helium

$$Y = 2X_n = 2 \frac{n/p}{1 + n/p}$$

so even without detailed calculations one sees that BBN can reproduce the observed He abundance, which mostly depends on n/p at the onset of D burning

detailed BBN predictions

Detailed nuclear reaction network + plasma properties coupled in a numerical code

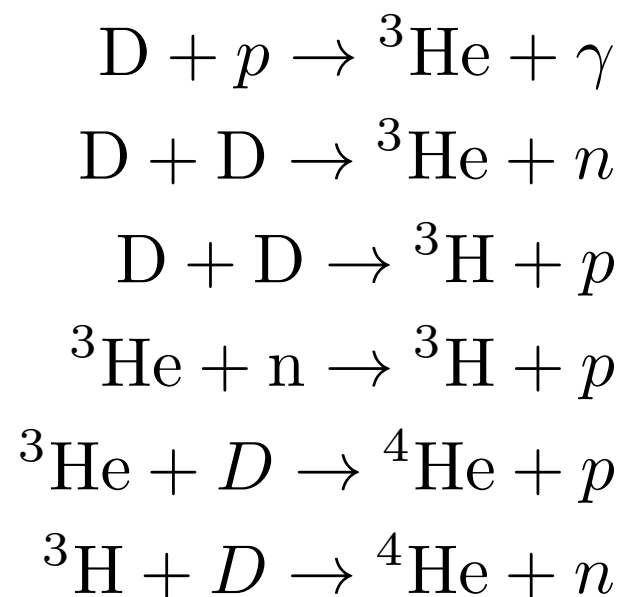


detailed BBN predictions

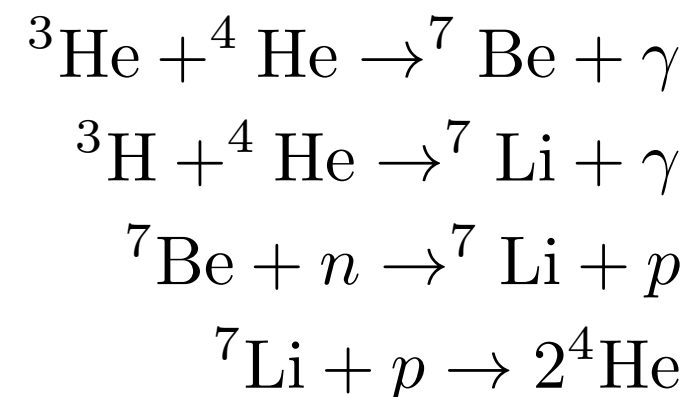
Final abundances determined by competition between the expansion rate H and the rates of the nuclear reactions involved

Most relevant reactions involve a strong interaction. The cross-sections of weak and EM reactions are too small, i.e. the timescales are large (especially for weak interactions which are completely negligible in BBN)

Helium



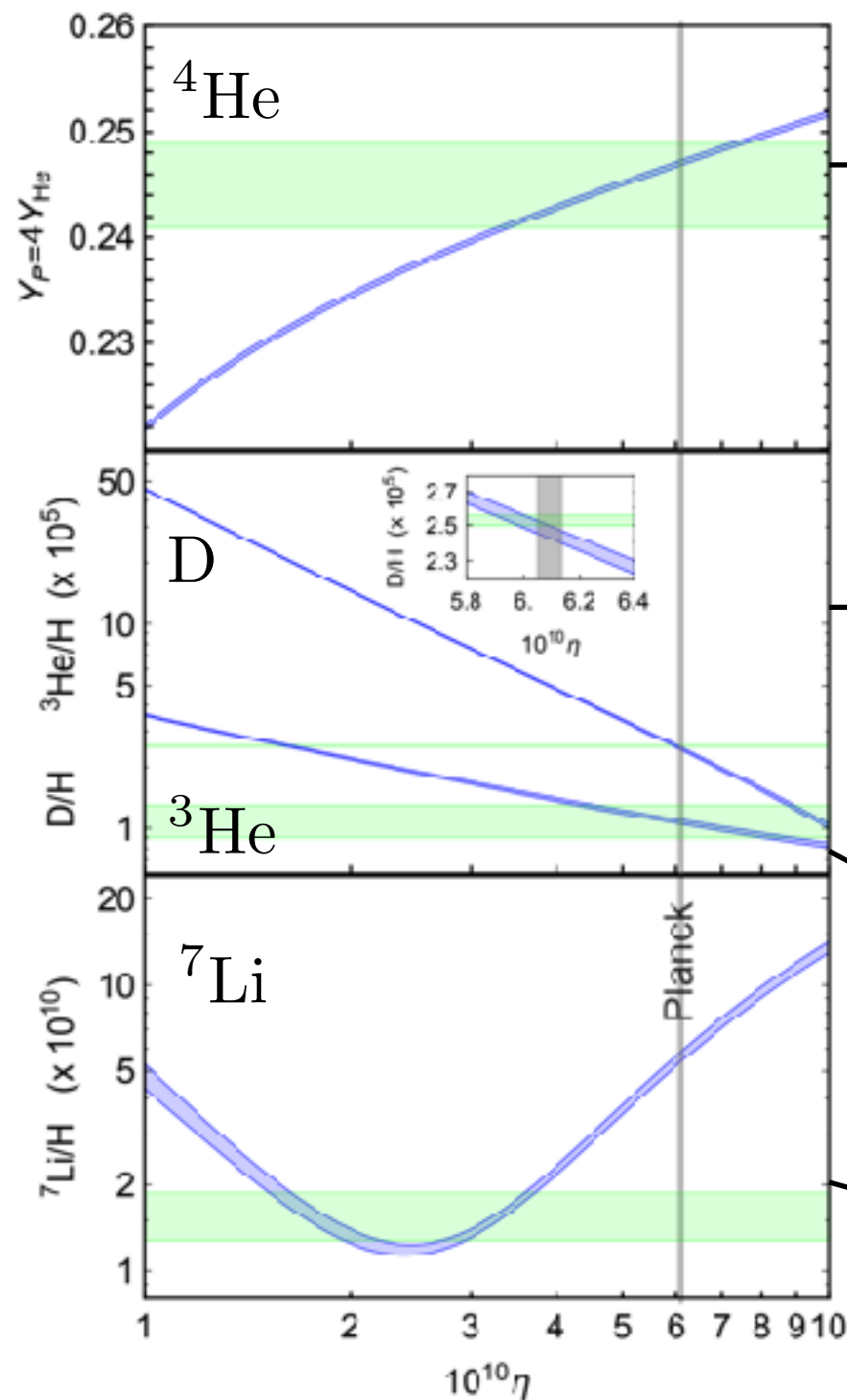
Lithium



There are no stable isotopes with mass number 5 (rates of proton and neutron captures on ${}^4\text{He}$ are far too slow), which halts the production of heavier nuclei. At the same time, temperature drops and all nuclear reactions stop

Variations of standard BBN

η as a free parameter



Helium abundances not very sensitive to η (almost all neutrons end up in helium nuclei anyways...)
Excellent agreement with observations

$$Y_p = 0.2449 \pm 0.0040$$

With increasing η , deuterium can more easily be converted to helium, so its residual abundance decreases. This sensitivity and the fact that D is not produced in stars, makes it an excellent “baryometer”. Again, there is perfect agreement with observations for the value of η derived by Planck

$$(D/H)_p = (2.53 \pm 0.04) \times 10^{-5}$$

Helium 3 is far more difficult to measure since its abundance is low and it is also created in stars

Lithium increases at low η (less protons around to destroy it), but also at high η (more beryllium produced which later decays to lithium)

The Universe has a lithium problem! 2-4 times less lithium than predicted

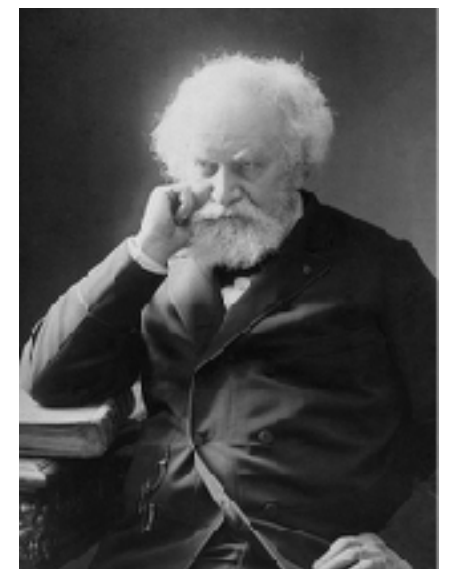
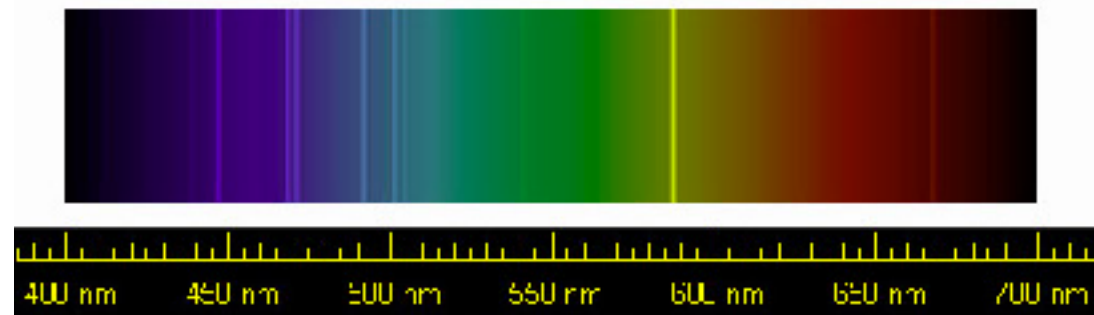
Observational constraints on BBN

To understand why there is a problem for Li while predictions for other elements seem to be OK, it is important to get precise observational constraints

Helium

Most stars (except at very high T) have no helium absorption lines in optical wavelengths (why?)

Sun abundance constrained by chromospheric emission



Pierre Janssen (1868)

Constraints mostly from emission lines in low-metallicity nebulae and galaxies.

Indirect constraints also come from its influence on stellar structure (e.g. asteroseismology, GC populations, etc)

$$Y_p = 0.2449 \pm 0.0040$$

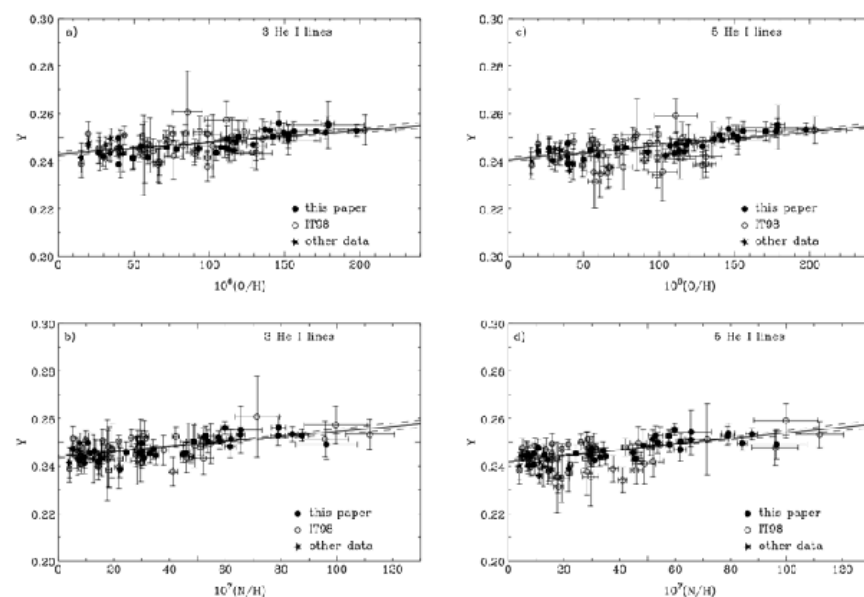


FIG. 2.—Linear regressions of the helium mass fraction Y vs. oxygen and nitrogen abundances for a total of 82 H II regions in 76 BCGs. In (a) and (b), Y was derived using the three $\lambda\lambda 4471, 5876$, and 6678 He I lines, and in (c) and (d), Y was derived using the five $\lambda\lambda 3889, 4471, 5876, 6678$, and 7065 He I lines.

Observational constraints on BBN

Deuterium

All deuterium must have originated during BBN since it is only destroyed in stars and it is vastly more abundant than lithium which can also result from spallation

D/H on earth is greatly enhanced by fractionation, hence, not representative of primordial abundance

Early constraints from solar wind, Moon, planets, gas-rich meteorites and C1 chondrites

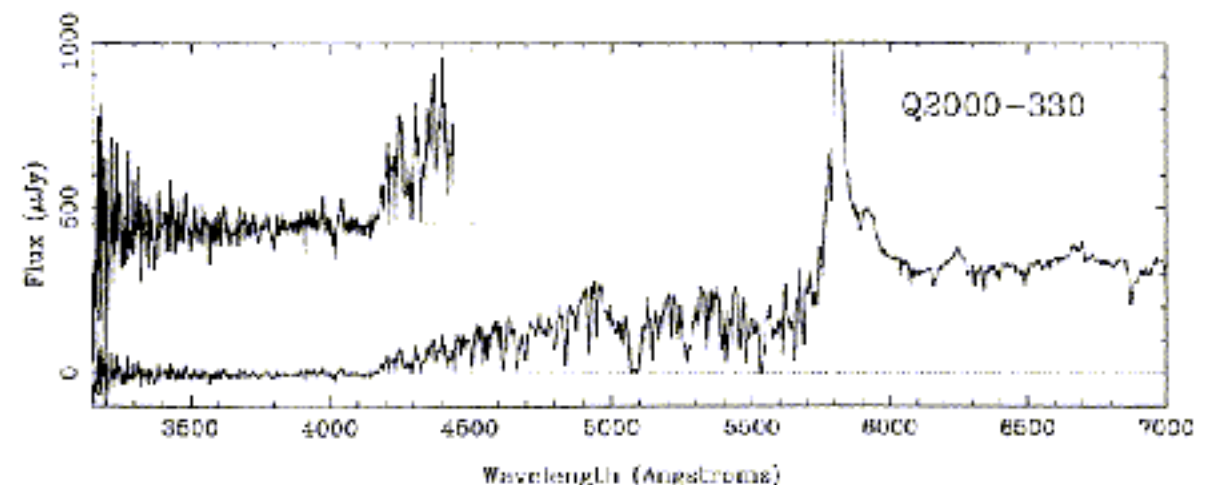
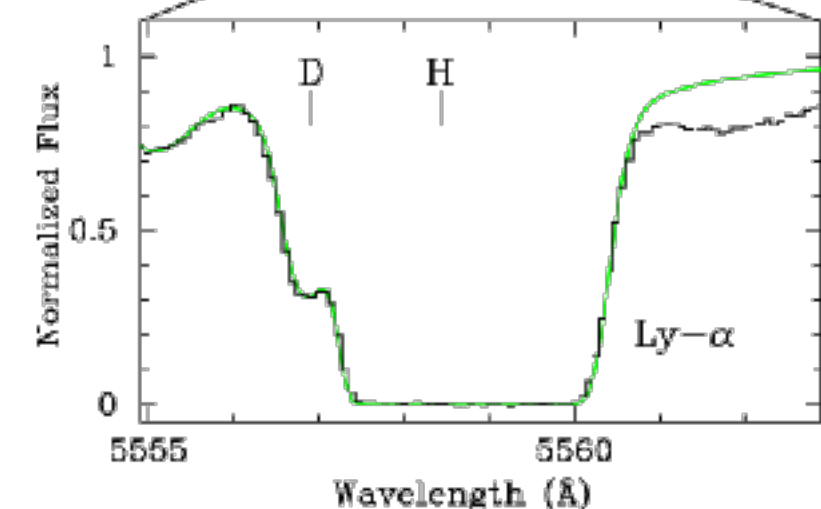
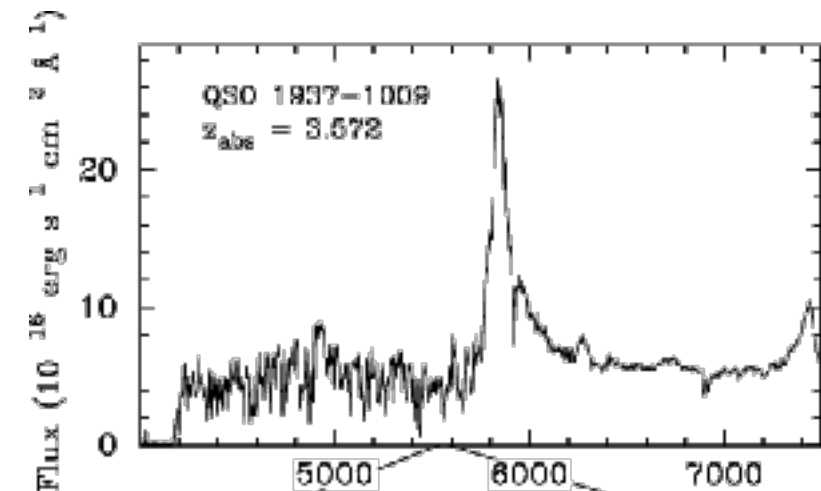
At radio wavelengths:
Transitions in DCN, DCH⁺, DI hyperfine structure

UV
HD and DI transitions in the ISM

With the advent of 10-m class mirrors, more direct constraints using the isotopic blue shift (87 km/s!) relative to H

$$(D/H)_p = (2.53 \pm 0.04) \times 10^{-5}$$

both in the ISM and in stars



Observational constraints on BBN

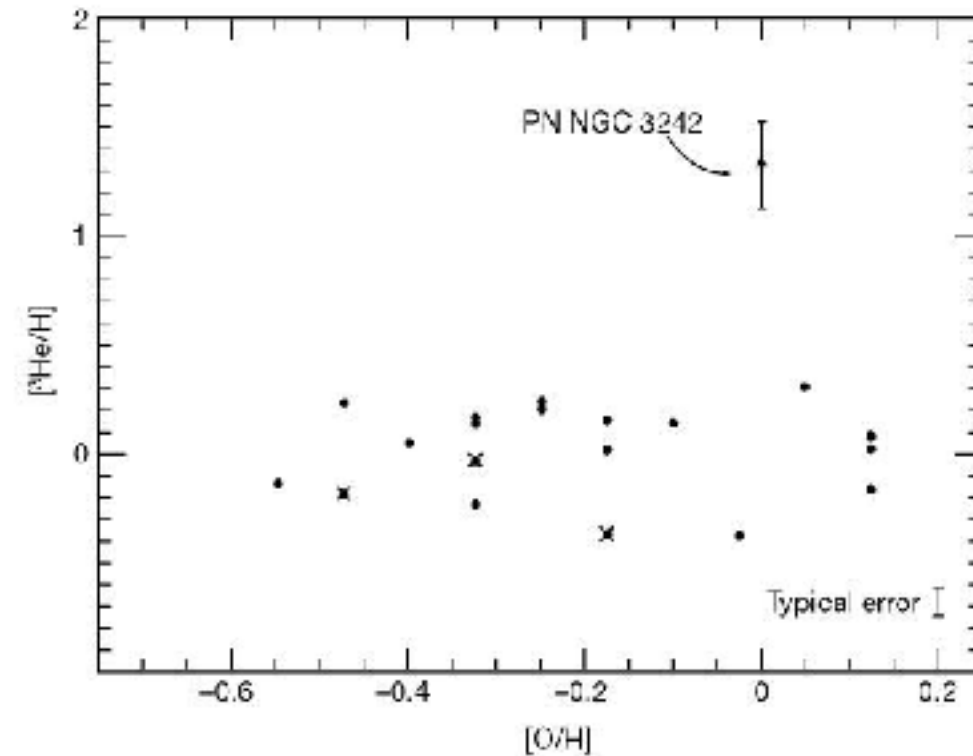
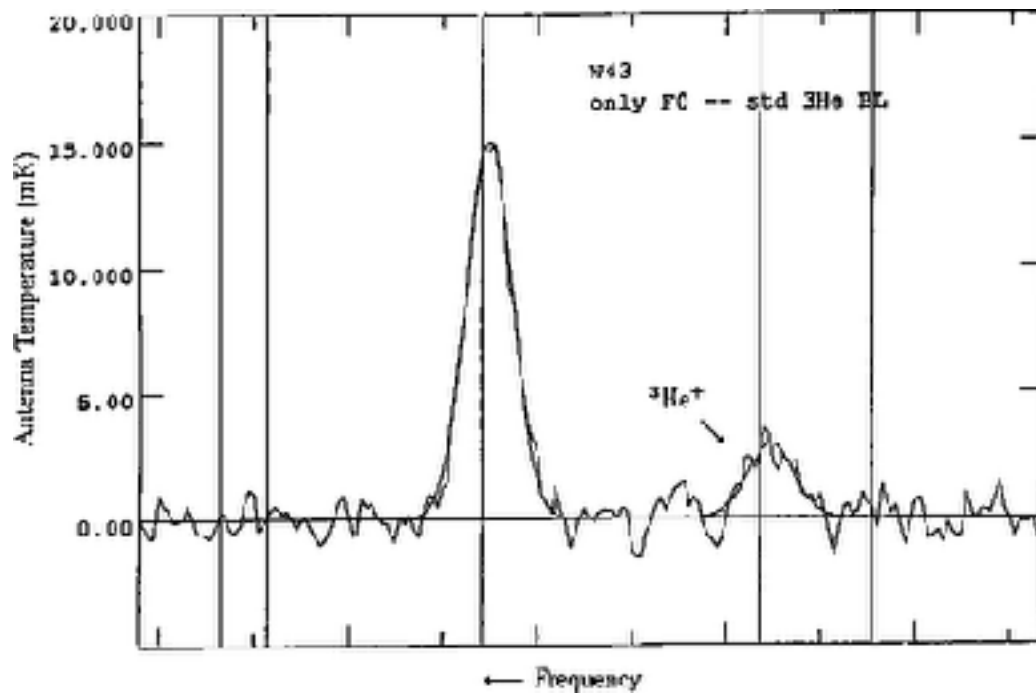
Helium-3

Extremely difficult to constrain primordial abundance for several reasons

Abundance lower than D, no absorption lines (similar to 4), atomic lines almost identical to Helium-4, also created and destroyed in stars

At radio wavelengths:

$^3\text{He}^+$ hyperfine transition at 3.46cm in “unevolved” objects (i.e. H II regions) but challenging due to background noise and competing emission mechanisms which require careful modelling



Observational constraints on BBN

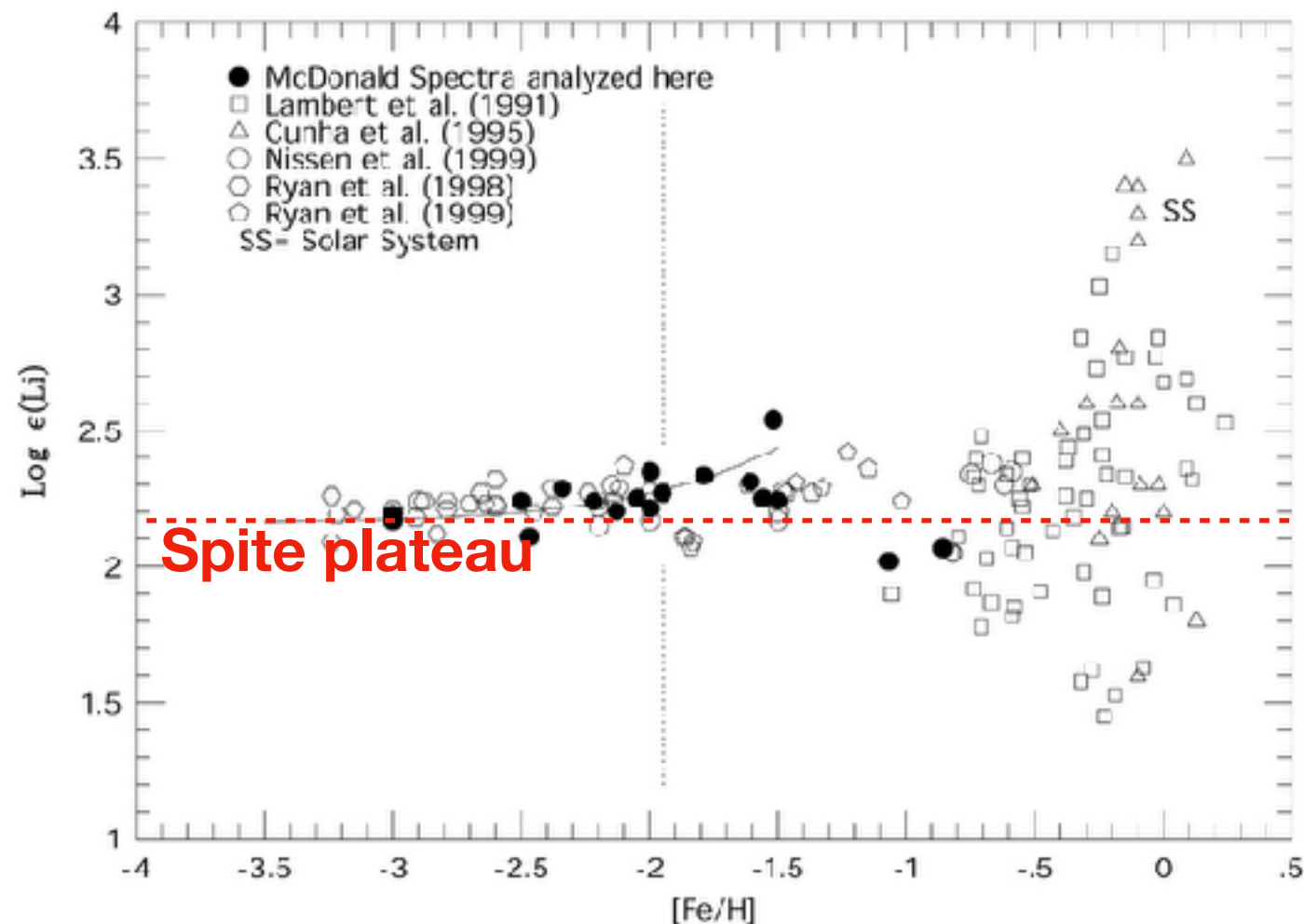
Lithium-7

Measurement of lithium in stars of different metallicities; extrapolated to $Z = 0$

In meteorites, most is in the form of Lithium-7 (92.5%)

In low-mass stars (including the Sun) with convective envelopes, lithium is depleted from the surface and destroyed. Abundance on the Sun is ~ 150 lower than “standard” theory predicts. Nevertheless, at low metallicities, all stars seem to have the same abundance, with very small scatter

The observed abundances are 2-4 times lower than standard BBN predictions



What is causing the lithium problem?

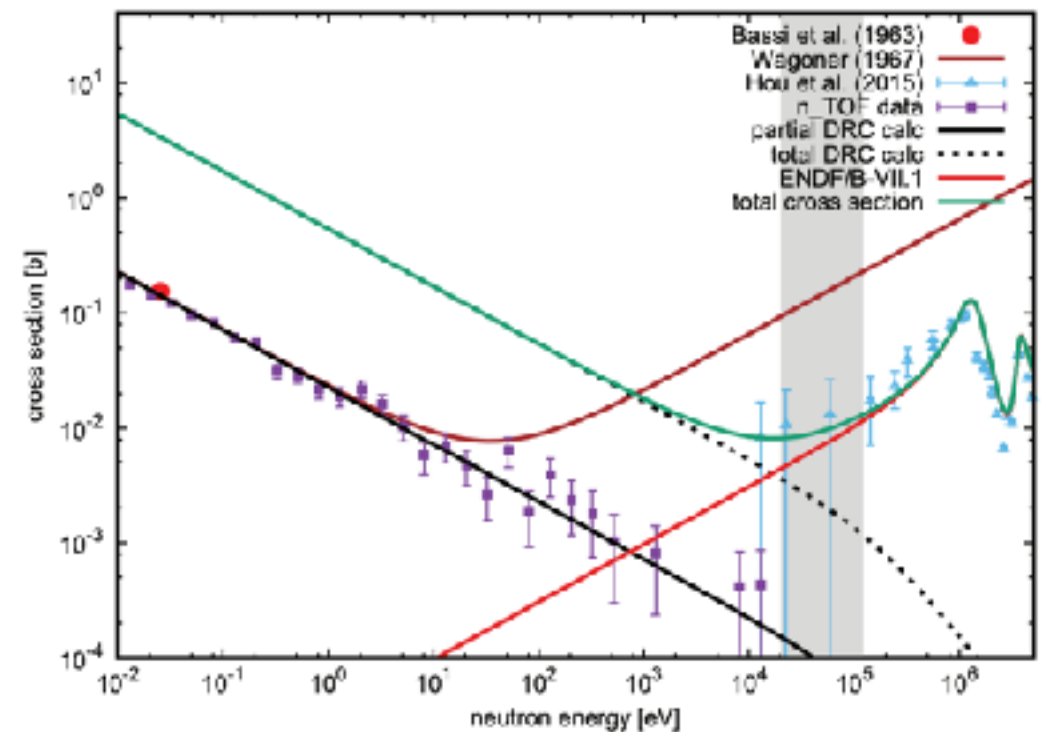
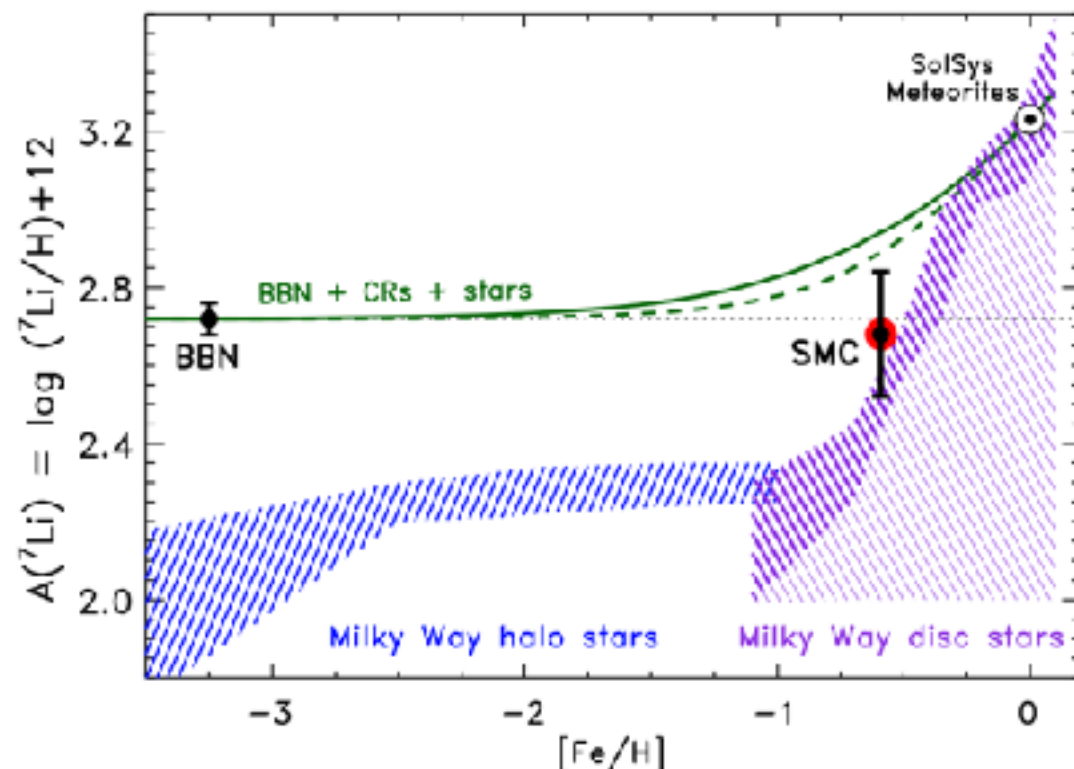
Unknown

Could be either due to observations, astrophysical variables (e.g. meridi), or new physics (unknown resonances). Exotic BBN is unlikely, as it is hard to change the Lithium abundance *only*

Some recent results

Cross section of ${}^7\text{Be}(n,\alpha){}^4\text{He}$, even lower than expected!

Barbagallo et al. (2016) PRL, 117, 15



SMC constraints on ${}^7\text{Li}$ are closer to BBN predictions
Howk et al. (2012), Nature, 489, 7414

BBN as a probe of cosmology and fundamental physics

Q: Why do we care about ultra-precise abundance measurements in the first place?

A: Because they can probe the conditions of the Universe well before the CMB era

We already discussed treating η as a free parameter

Then, constraints on primordial D/H yield an (independent) constraint on η , and through that, on the baryon density Ω_m (exercise 3.3)

$$\eta_D = 2.73 \times 10^{-8} \Omega_b h^2 \Rightarrow \Omega_b h^2 = 0.0222 \pm 0.0003$$

BBN predicts an extremely small baryon density (the very reason it was dismissed in the 50s...), which is consistent with the CMB constraints

What else?

We can also treat the number of neutrino families as a free parameter

$$\rho_{\text{rad}} = \rho_\gamma + \rho_{e^\pm} + N_\nu \rho_\nu := (g_\star/2) \rho_\gamma \Rightarrow T = \left(\frac{3c^2}{16\pi g_\star a G} \right) t^{-1/2}$$

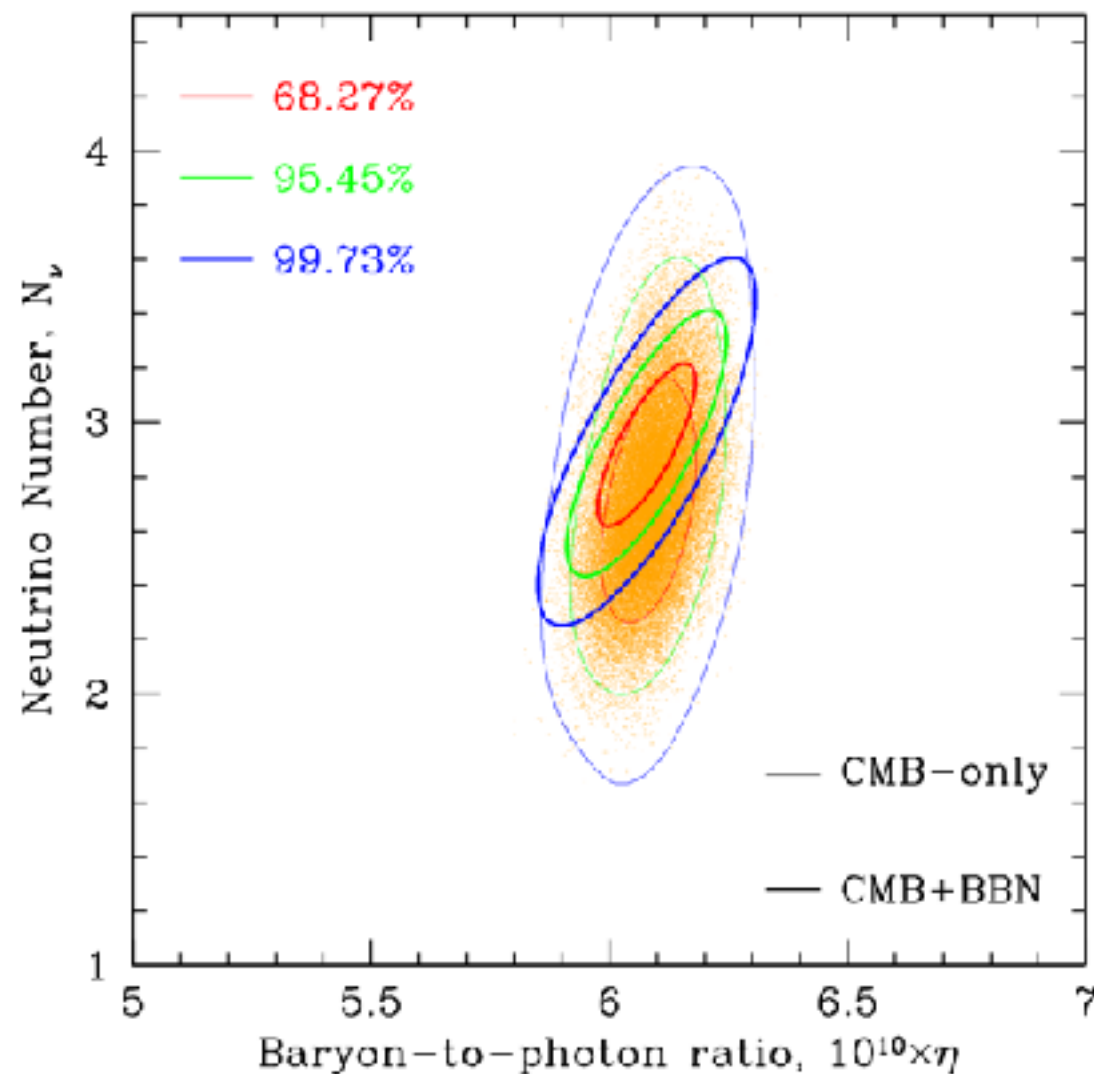
This affects the energy density vs T and therefore the expansion rate vs time, the freeze out value of n/p and the temperature T_p at which it occurs \rightarrow Helium abundance affected

BBN as a probe of cosmology and fundamental physics

The expansion rate varies with $\rho^{1/2} \propto \left(\frac{11}{4} + \frac{7}{8}N_\nu\right)^{1/2} T^2$, while for n-p reactions equilibrium is broken at ~ 0.8 MeV, $e^{-1.29 \text{ MeV}/0.8 \text{ MeV}} \simeq 0.2$

The precise value of the critical temperature T_d for decoupling, scales as $T_d^2 \propto \tau_{1/2} \left(\frac{11}{4} + \frac{7}{8}N_\nu\right)^{1/2}$

Joint constraints on η , N_ν

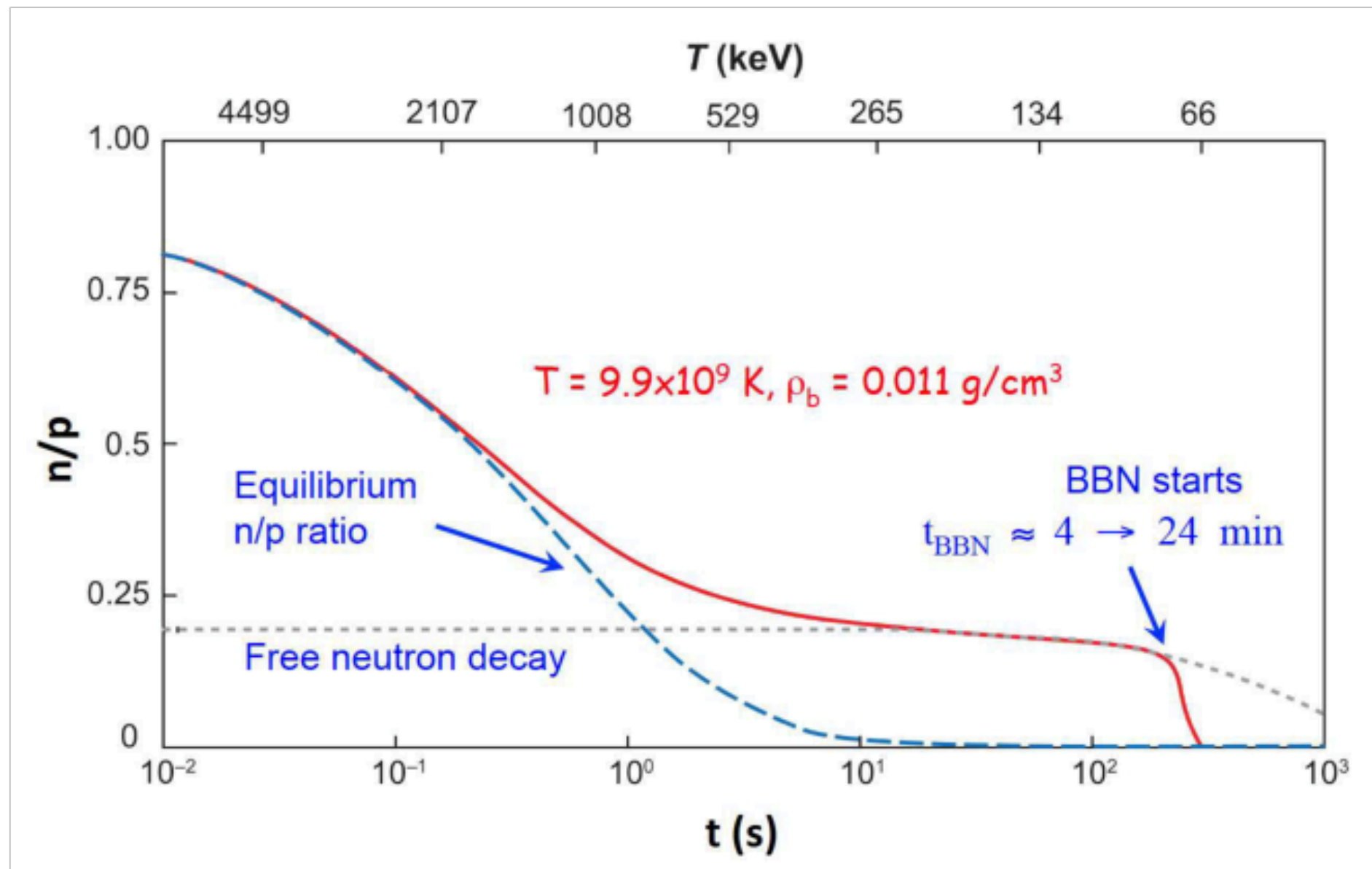


The physics describing the Universe at MeV scales are consistent with the physics at eV scales

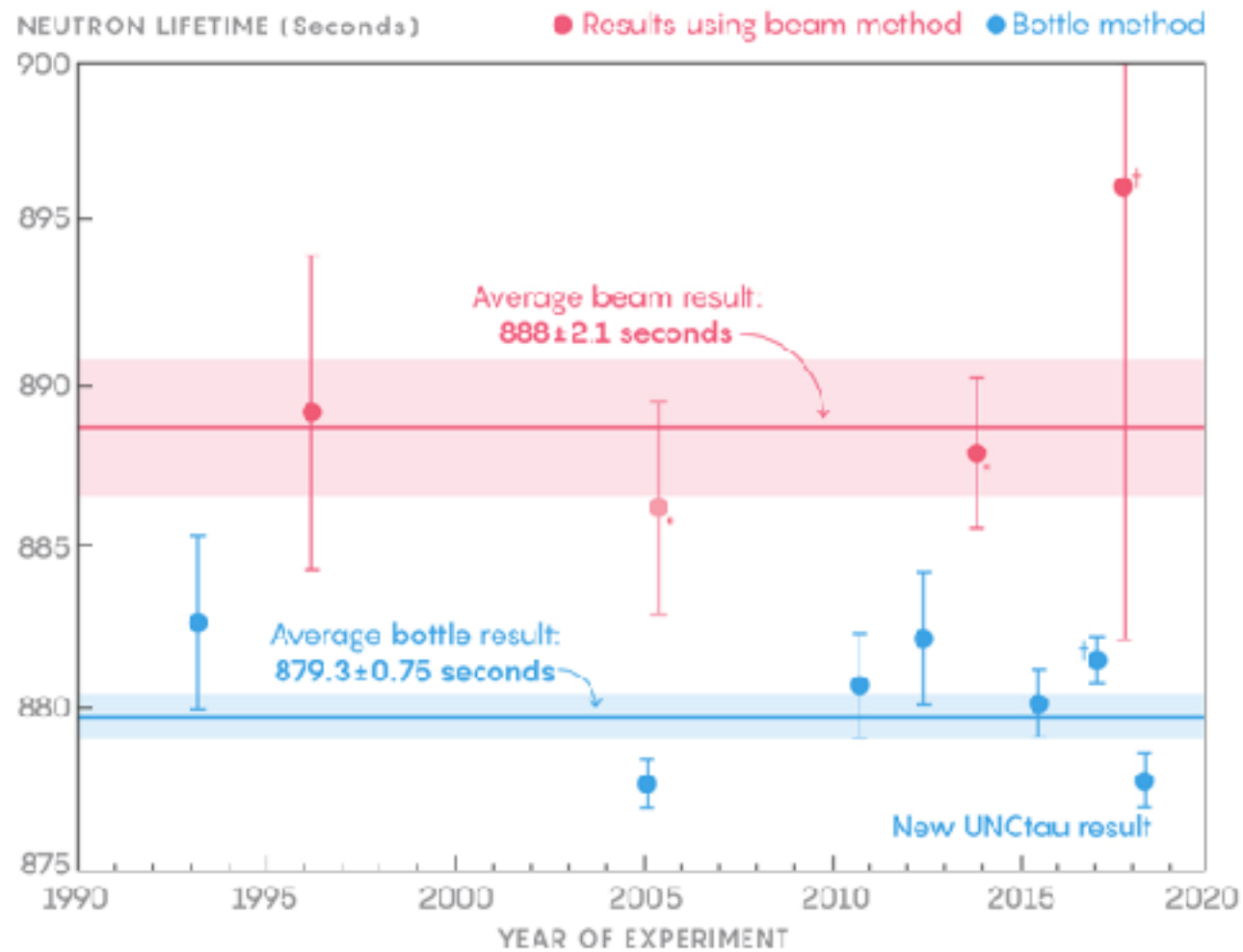
BBN as a probe of cosmology and fundamental physics

Other possibilities

Deviations from GR affecting the expansion rate, weak interaction physics (affecting initial abundances), neutron lifetime, alternative reaction rates....

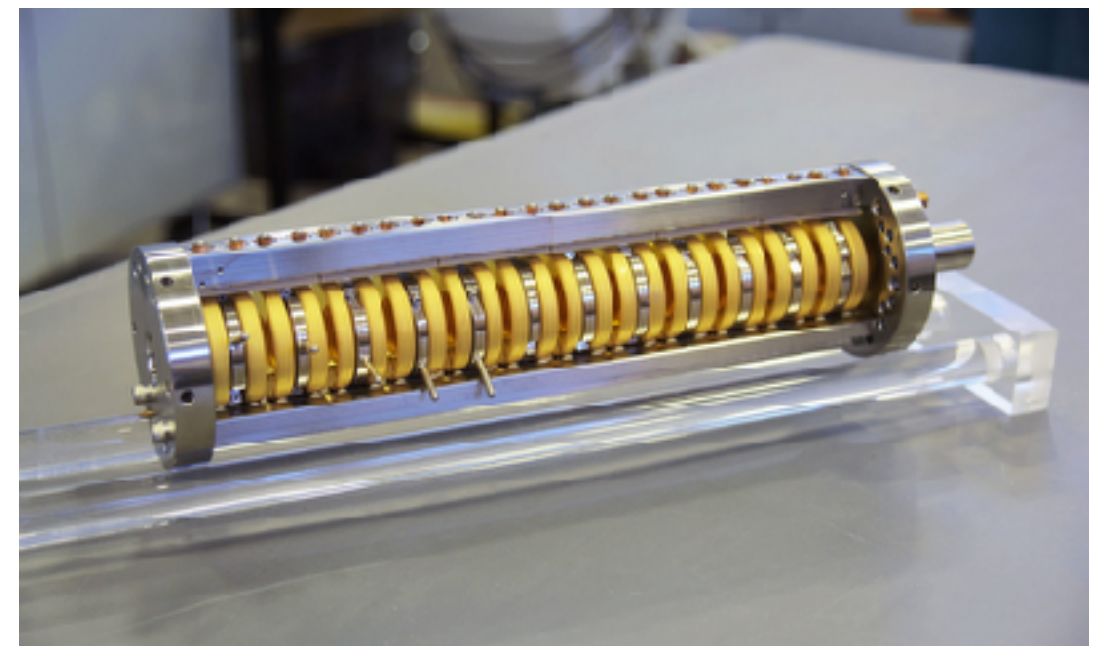


Neutron decay



Thus far, the measured mean lifetime of neutrons seems to depend on the measurement method.

Could be due to new physics! What is the effect on BBN?



Can we see the Universe before the onset BBN?

An interesting recent result

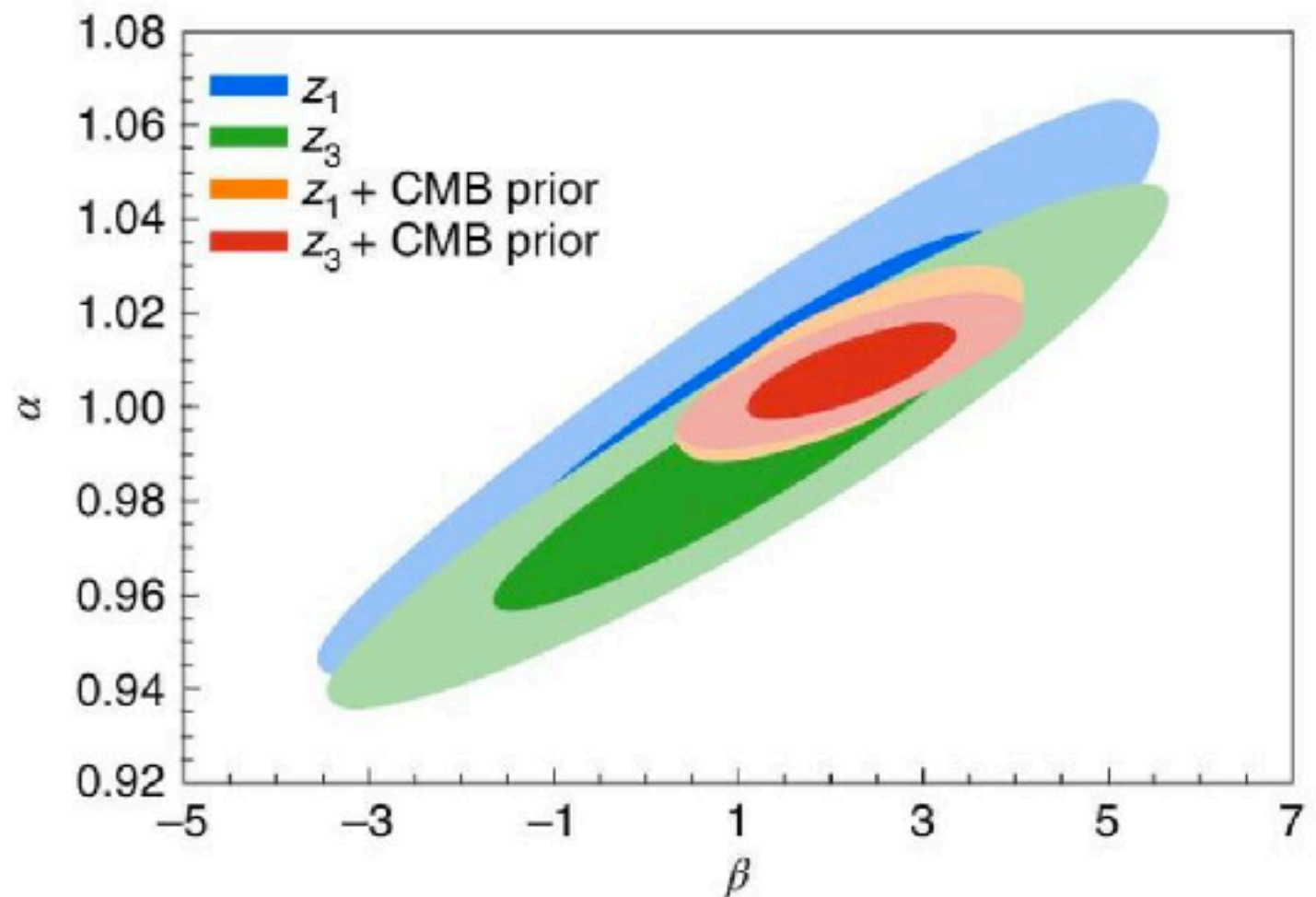
Article | Published: 25 February 2019

First constraint on the neutrino-induced phase shift in the spectrum of baryon acoustic oscillations

Daniel Baumann, Florian Beutler, Raphael Flauger, Daniel Green , Anže Slosar, Mariana Vargas-Magaña, Benjamin Wallisch & Christophe Yèche

Nature Physics **15**, 465–469 (2019) | [Download Citation](#) 

**First (indirect) detection
of the cosmic neutrino background!**



Take Home Message

1. Big bang nucleosynthesis predicts the abundances of light isotopes extremely accurately, **over 9 orders of magnitude**
2. Lithium abundance remains challenging
3. Inference of primordial abundances from observations is challenging
4. BBN probes the very early Universe, well before the CMB was emitted
5. As precision in measurements increases, we will be able to constrain physics beyond the standard model and GR with increasing precision

Coming up

Hydrogen burning in (and on) stars (spoiler: quite different than BBN)

